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OF THE
NATIONAL BUREAU OF STANDARDS

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RESEARCH PAPER RP759

Part of Journal of Research of the National Bureau of Standards, Volume 14, February 1935

THE NATIONAL PRIMARY STANDARD OF RADIO FREQUENCY

By Elmer L. Hall, Vincent E. Heaton, and Evan G. Lapham

ABSTRACT

The primary frequency standard consists of two independent groups of piezo oscillators. Section I consists of four piezo oscillators with frequencies of 100 kc/s. Section II has two piezo oscillators, one with a frequency of 100 kc/s, the other 200 kc/s, from which a 100-kc/s output is obtained by means of a submultiple generator. The absolute frequency of one of the units of each section is checked daily against Arlington time signals by a synchronous-motor clock driven by the 100th submultiple of the frequency of the controlling unit. Daily measurements provide a check on the frequency variations of the standard over an extended period. Frequency variations over short intervals are shown by an automatic recorder of the frequency difference between one of the units of section I and each of the other five units.

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I. INTRODUCTION

Among the functions of the National Bureau of Standards as given by the act under which it was established are the maintenance and improvement of the various national standards and the testing and calibration of measuring apparatus. The Radio Section of the Bureau has as one of these functions the carrying out of the above activities with respect to the national primary standards of radio frequency. The Bureau has had to improve the apparatus and increase the precision and the accuracy of the primary frequency standards to meet the increasingly rigorous demands of scientific and engineering advances. The primary frequency standards have taken different forms during the last decade, i. e., standard wave-length circuits, vacuum-tube driven tuning forks and associated harmonic amplifiers, simple temperature-controlled piezo oscillators, and, finally, the more complex piezo oscillators maintained under the most constant conditions such as are essential if the great precision desired is to be realized. During the development of the piezo oscillator to the important place it holds today as a radio-frequency standard, the Bureau has enjoyed the cooperation and assistance of several research laboratories and instrument manufacturers conducting similar work.

The present primary standard [1] contains two independent groups of piezo oscillators designated sections I and II. Section I consists of four piezo oscillators with fundamental frequencies of 100 kc/s. Section II has two piezo oscillators, one with a fundamental frequency of 100 kc/s and the other 200 kc/s, with a submultiple generator controlled from the latter which provides the desired 100-kc/s output.

The absolute frequency of one unit of each section is checked daily with the Arlington time signals by means of synchronous-motor clocks. The voltages which control the synchronous motors are an exact 100th submultiple of the frequencies of the controlling standards. The daily measurements provide a check on the frequency variations of the standard over an extended period. The variations over short-time intervals are indicated by an automatic recorder of the frequency difference between one of the piezo oscillators in section I and the other five units of sections I and II.

II. DESCRIPTION OF PRIMARY STANDARD

In 1929 the Bureau acquired section I of the primary standard as developed by the Bell Telephone Laboratories. It is shown in figure 1.

As this apparatus has been described (1)[2] in detail by the designer, only a brief description will be given here. A specially cut toroidal quartz plate is used in order to obtain a small temperature coefficient of frequency. This quartz toroid is mounted on a horizontal rod of fiber passed through the hole. Disks of aluminum at either end of the fiber rod serve as electrodes. The spacing of the electrodes is kept constant by a section of pyrex glass tubing, the ends ground flat and parallel, placed between them. The plate holder is wrapped in felt, ½ cm thick, and placed within a thick-walled cylinder of aluminum. On the outer surface of this cylinder heater coils are wound for maintaining the temperature of the quartz plate constant. Alternate layers of felt and copper are placed over the heater coils to serve as heat attenuation. A thermometer and a mercury thermoregulator for controlling the temperature are placed within the wall of the aluminum cylinder. This temperature-controlled chamber is mounted on a brass plate with a bell jar sealed over it. The air pressure is maintained about 5 cm of mercury below atmospheric pressure. The oscillator and amplifier circuit arrangements are located beneath the brass plate. Each unit is mounted on damped springs in a temperature-controlled cabinet, as shown in figure 1.

There are a number of factors which govern the constancy of the frequency of the piezo oscillator. Among these are temperature, pressure, humidity, and voltages applied to the oscillator tube. The choice of dimensions and orientation of the quartz plate makes the temperature coefficient of frequency of the order of 1 part in a million per °C (1) as compared with 20 to 30 parts in a million for a solid quartz disk of the same frequency. The variation of temperature at the quartz plate due to the periodic operation of the heater control is less than 0.001° C (1), which is small enough to produce a negligible variation in frequency.

[1] Primary standard will be used hereafter meaning primary frequency standard.
[2] The figures given in parentheses here and elsewhere in the text refer to the numbered references at the end of this paper.

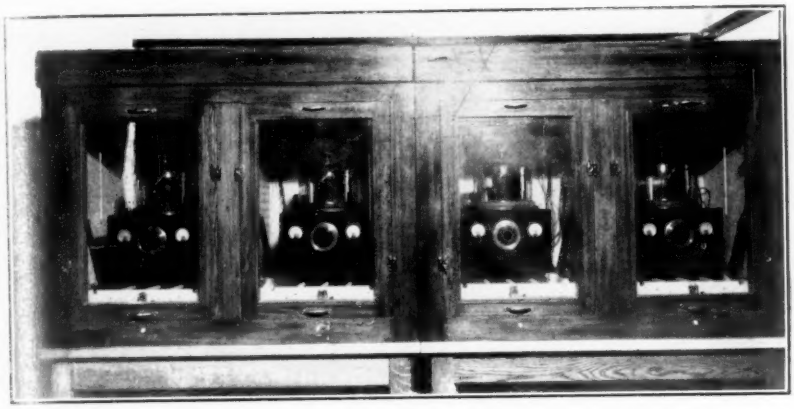

FIGURE 1.—*The four piezo oscillators of section I of the primary frequency standard mounted in temperature-controlled cabinets.*

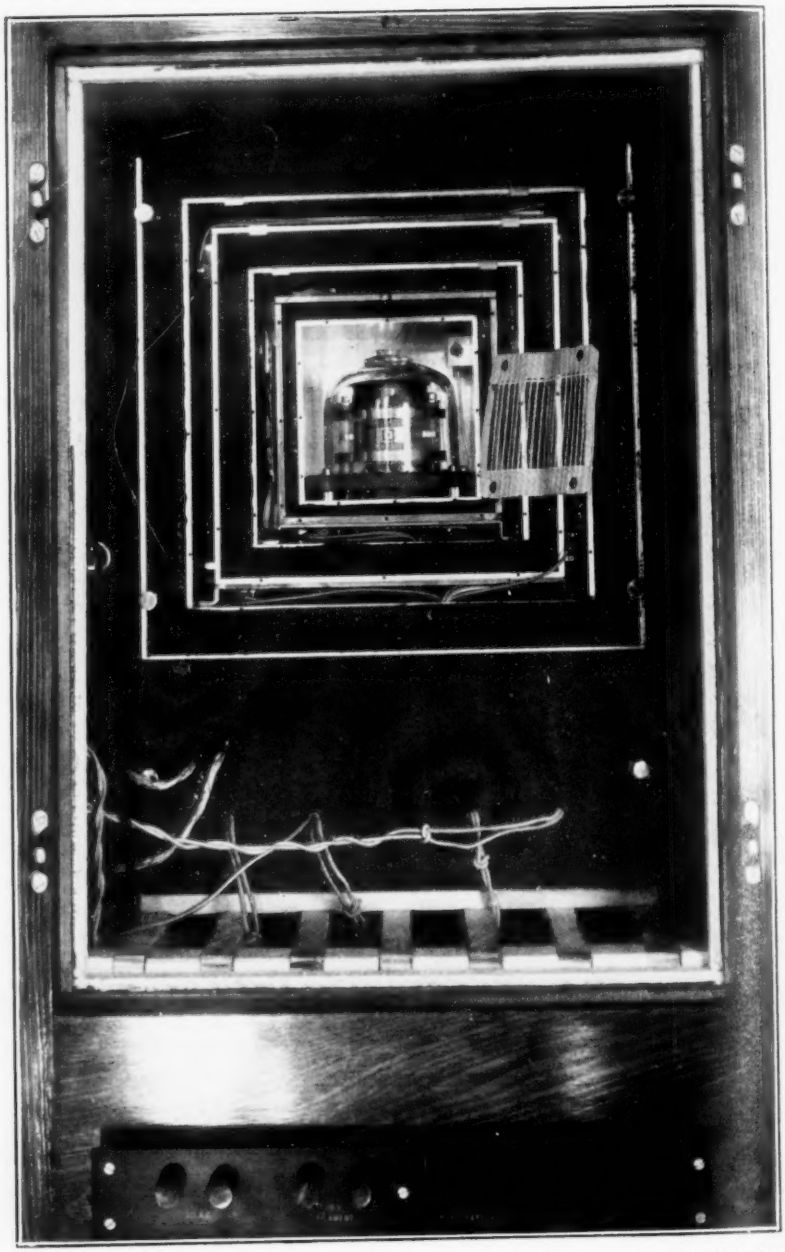

FIGURE 2.—*Primary no. 25 showing construction of temperature-controlled boxes.*

To protect the quartz plate from humidity and pressure variations it is kept under a bell jar at a pressure slightly below atmospheric. Considerable difficulty was experienced in finding a wax suitable to seal the glass bell jar to the brass plate and maintain the joint airtight over considerable periods of time. The wax used at present is made by combining crude rubber, vaseline, and beeswax at a temperature slightly higher than the melting point of the rubber. The exact proportions of the various constituents depend on the temperature to be maintained on the brass plate. Any seasonal variation in temperature in the outer temperature-controlled cabinet produces a change in pressure under the bell jar due to the change in temperature of the air in the jar. This variation is quite small with the present temperature control. The pressure within the bell jar continually decreases as indicated on the closed-tube manometer during the normal operation of the piezo oscillator.

There are three ways of adjusting the frequency of these piezo oscillators to 100 kc/s. A coarse adjustment may be made by changing the air gap in the plate holder by turning a central threaded portion of one electrode. A somewhat finer frequency adjustment may be made by changing the air pressure under the bell jar. An increase in pressure of 1 cm of mercury produces an increase in frequency of 1 part in 10 million. The finest adjustment may be made by means of a small cylindrical condenser, having a maximum capacity of about 5 $\mu\mu$f, shunting the electrodes of the plate holder. An adjustment of one division on the dial of this condenser corresponds to a change of frequency of about 1 part in 100 million (1). The total adjustment changes the frequency 1 part in 100 thousand.

Each piezo-oscillator circuit arrangement is of the familiar type in which the quartz plate is connected between the grid and filament of the oscillator tube and a tuned circuit is connected to the plate. The constants of this tuned circuit are such as to produce the least variation in frequency with temperature variation or aging of the circuit. Three 100-kc/s amplifiers provide independent output voltages to control the submultiple generator, to compare with the reference oscillator, and for other frequency measurements.

The submultiple generator is a series of multivibrators and amplifiers, which provide voltages at 10,000 and 1,000 cycles per second. These frequencies are exact submultiples of the frequency of the controlling standard. Three output amplifiers are provided on both the 10- and the 1-kc/s stages.

If the piezo oscillator has a frequency of exactly 100 kc/s, the frequency of 1,000 cycles per second will drive a synchronous motor geared to the hands of a clock at such a rate as to keep correct time. On the shaft of the synchronous motor are generators of 100 and 10 cycles per second, which are therefore submultiples of the original piezo-oscillator frequency. The generators are connected to amplifiers so that these frequencies are also available for use in the laboratory.

At the present time section II of the primary standard has but two piezo oscillators built a year apart in the course of experimental development and study of piezo oscillators. These two units differ in design from each other and from the four units described above. In the two years that the one unit has been in use, valuable performance data have been obtained, which have been useful in the design of other piezo oscillators now under construction.

The older of the two piezo oscillators, shown in figure 2, served as a design unit from which an improved standard model was developed. The circuit arrangement used is similar to that described above. The 100-kc/s circular quartz plate is clamped between 3 pointed screws bearing 120 degrees apart in a V-shaped groove on the cylindrical surface of the disk. All metal parts of the holder were made of stainless steel, and a pyrex glass was used for insulating parts. The quartz-plate mounting was placed under a bell jar with its edge ground flat to fit a pyrex glass plate serving as a base. A

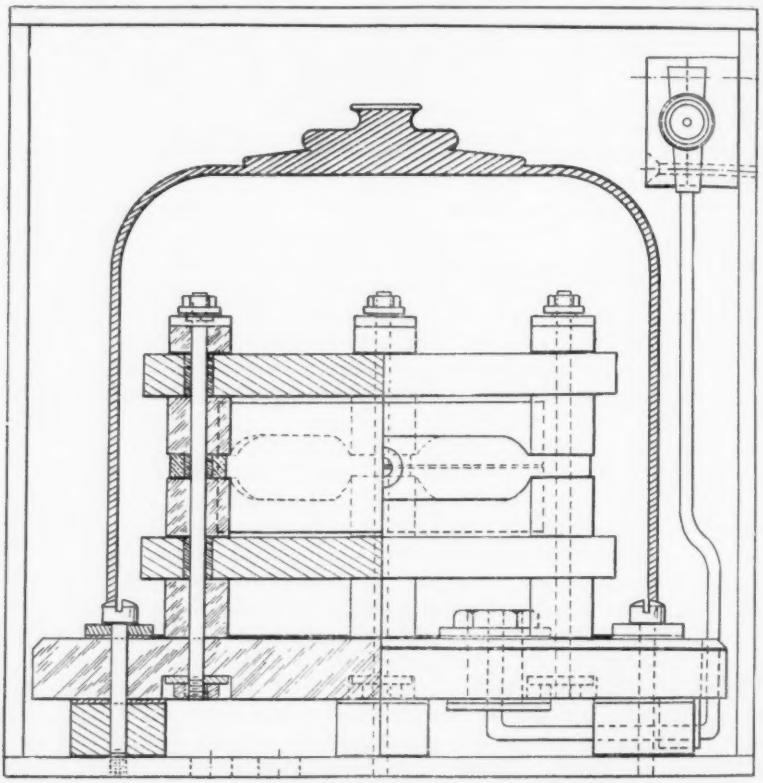

FIGURE 3.—*Assembly of quartz plate and mounting within bell jar, primary no. 25.*

small copper tube and valve were provided and all openings sealed so that the bell jar could be evacuated. The pressure was reduced to ½ atmosphere. No difficulty was experienced with air leakage. A sectional drawing of the quartz-plate assembly is shown in figure 3. The frequency of the quartz plate decreases 1 part in 10 million per volt increase in plate potential. The variations in the voltage on the plate of the oscillator tube were minimized by using a type-874 voltage-regulator tube.

The second unit of section II is still in an experimental state, not conforming to the standard design which has been evolved. However, its frequency characteristics were found to be quite satisfactory. This unit was the result of an uncompleted study of the method of cutting and mounting toroidal quartz plates in a clamped holder as applied to the resistance-capacity type of circuit arrangement, the

quartz plate being connected between the plate and grid of the tube. A circuit diagram of the piezo oscillator is given in figure 4. A mathematical study of this type of piezo oscillator by Terry (2) showed that changes in filament or plate voltages or in other circuit elements such as might be experienced in normal operation produced only slight frequency changes. This type of piezo oscillator therefore seemed particularly suitable for use as a standard. A 200-kc/s, 30-degree circular quartz plate was cut and portions removed until it resembled a small wheel with heavy outer rim and a small central hub held in place by two thin spokes. The hub was drilled on both sides, leaving a small web in the center. The approximate dimensions

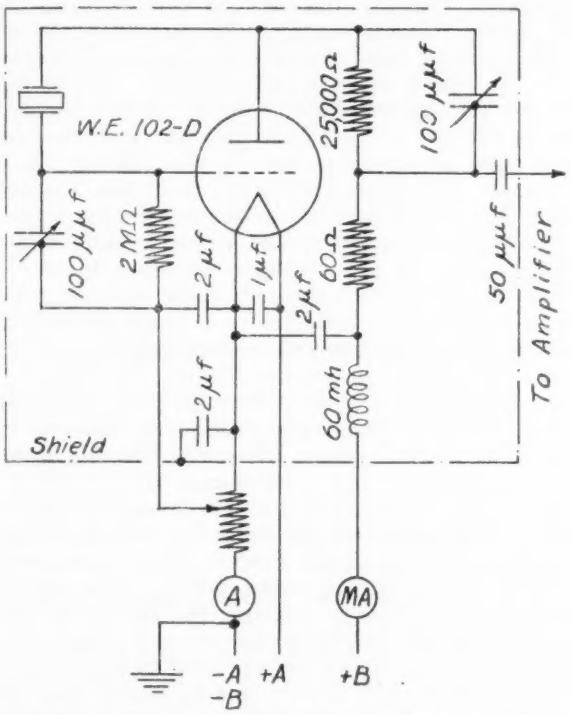

FIGURE 4.—*Electrical circuits of primary no. 26.*

of the plate were, outside diameter 37 mm, inside diameter 26 mm, and thickness 10.5 mm. The wheel was mounted on short metal rods fitting into the holes. Circular electrodes were held in position on either side of the wheel. No pressure control was provided in this unit. The output of this type of piezo oscillator is quite feeble, requiring more amplification than with the more usual type. Other disadvantages are the fragility and difficulty of preparing this type of quartz plate. Its improved frequency stability tends to compensate for these disadvantages. For example, an increase of 100 percent in the plate voltage applied to the oscillator tube in this type of piezo oscillator changed the frequency less than 1 part in a million. Large changes in the load resistance, plate condenser, and grid condenser gave relatively small frequency changes. Hence changes in the circuit elements and voltages applied, such as may be expected in the

course of usual operation, are almost without effect on the frequency of this piezo oscillator. The mounted quartz plate used has a temperature coefficient of frequency of about 1 part in a million per °C.

The power supply is quite a problem due to the numerous tubes of the various units. Section I requires 700 watts of power and section II 400 watts continuously, and additional power is needed intermittently for supplying heat to the piezo oscillators and for operating the various motors and relays.

Each section of the primary standard has a separate power supply. Working batteries of 18 and 135 volts supply the filament and plate voltages, respectively. Stand-by batteries of 26 and 190 volts are connected through variable resistors to the working batteries for filament and plate voltage supply. All batteries are of the lead-acid type. A marginal relay connected across each working battery operates a motor which adjusts the charging rate to maintain the working battery voltage constant to better than 1 percent.

When section I of the primary standard was first installed, a motor-generator was used to supply power for maintaining the batteries at constant voltage. The motor operated from the 3-phase, 220-volt, 60-cycle power lines. Because of the large variation in the load at the Bureau, the voltage of the power line varied greatly and produced a corresponding change in the voltages supplied by the generators. The two sets of batteries with the automatic adjustment of charging rate smoothed out the fluctuations to a large degree. Frequently the voltage supplied by the generators decreased to such a value that, when the protective relays failed, the battery discharged through the generator, reversing its field and at times burning out a relay winding.

In the fall of 1931 the motor-generator was replaced by full-wave rectifiers operating from 3-phase, 220-volt, 60-cycle lines. Copper-oxide rectifiers supply the plate power and tube rectifiers supply the filament power. No further trouble has been experienced with the power supply. The power-supply units for section II are similar to those for section I. The direct current from each rectifier passes through a filter circuit arrangement which, with the two sets of storage batteries, is effective in reducing the 360-cycle pulsations to a negligible amount before reaching the primary standard units.

A separate 24-volt storage battery is used to supply heater current for the temperature control of the quartz plates and for operating relays such as those used to control the operation of the outer heaters for which the 60-cycle lines supply power. This battery is charged from the 110-volt direct-current lines which are sufficiently constant in voltage for the purpose.

It is quite important to keep both the filament and plate voltages of the primary standard constant, because voltage fluctuations will introduce frequency changes of different amounts for the different units. The two marginal relays used to maintain constant voltage for section I of the primary standard are similar. A solenoid connected in shunt with the battery whose voltage is to be controlled is mounted with its axis vertical. An iron armature is placed in the solenoid so that its weight is balanced by the force exerted by the field within the coil. Whenever the voltage of the battery decreases, the armature moves downward to close a contact and adjust the motor-driven rheostat to increase the charging rate. An increase in voltage moves the armature upward to close a contact and decrease the charging rate.

The system of voltage regulation in section II originally had two voltage relays, one for the filament and the other for the plate voltage. The principle of operation was essentially the same as that of the marginal relays in section I. The relays had an energized coil balanced in the field of a fixed magnet against the pressure exerted by a helical spring. As the coil moved, one of the two connections was made, depending upon whether the voltage was excessively high or low. These contacts, through a single-tube amplifier, operated a series of two relays which controlled the motor-driven rheostat. The arrangement was unsatisfactory on account of sticking of the contacts and sluggishness in the movement of the coil due to friction in the bearings.

A voltage-divider circuit arrangement was then used, in which the controlled plate voltage was the working battery and a UX874 voltage-regulator tube, connected through a current-limiting resistor across

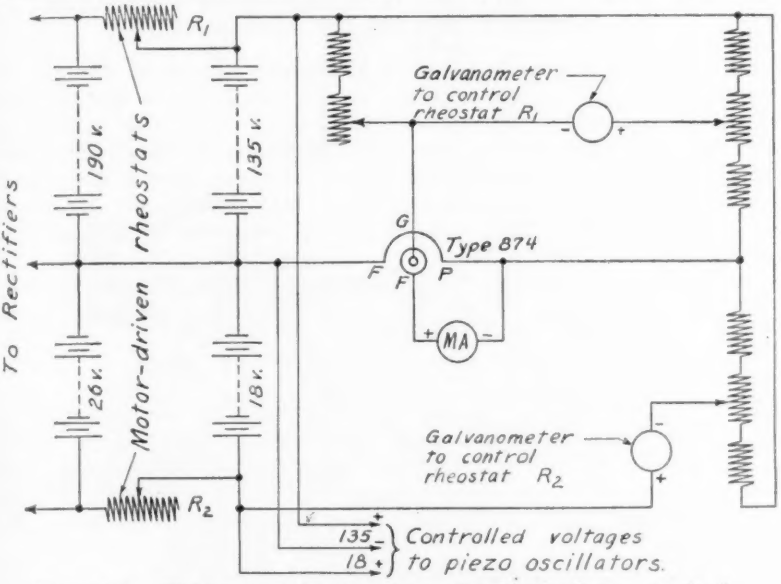

FIGURE 5.—*Voltage control system for section II of primary standard.*

the working plate battery, provided a reference-standard voltage of approximately 90 volts. The voltage relays were changed to serve as zero-center galvanometers by shorting the series resistance and readjusting the spring so that the coil rested in the midposition with no current flowing. This galvanometer, connected in the customary position in a potentiometer circuit, kept the circuit balanced by adjusting the working plate battery voltage. The filament voltage was controlled in a similar manner, except that the working plate battery voltage was used as the reference voltage. The difficulty due to bearing friction was overcome by superimposing a small 60-cycle voltage upon the current through the galvanometer. It was still necessary to clean the contacts at frequent intervals.

These zero-center galvanometers have been replaced by galvanometers with motor-driven contactors. These units made it possible to eliminate the amplifier and a relay, and provided positive operation of the single relay which controls the motor-driven rheostat This arrangement readjusts the charging rate once in 10 seconds if needed,

by operating the rheostat one second in the direction dictated by the instrument balance. The circuit arrangement used is shown in figure 5. This system of automatic voltage control gives somewhat better regulation than the system used in section I because it is not affected by changes in the room temperature, has higher sensitivity, and fewer parts that are liable to failure. It has the disadvantage, however, that it does not respond as quickly to large changes in the power-supply voltage. If a power failure occurs, it requires about two minutes for this system to reach a balance, whereas the system in section I will make the full adjustment in one operation of the rheostat.

All of the various parts of the automatic voltage control units are operated on direct current supplied by the heater batteries.

III. METHODS USED IN DETERMINING FREQUENCY AND DESCRIPTION OF APPARATUS

In order to calculate the absolute frequency of one of the units of the primary standard, the number of vibrations of the quartz plate in a known interval of time must be known. The time interval of 24 hours is determined by radio time signals from the U. S. Naval Observatory. The number of vibrations of the quartz plate is indicated by the synchronous-motor clock controlled by the frequency standard under observation. Each second indicated by the synchronous-motor clock requires 100,000 vibrations of the quartz plate. The frequency f, in cycles per second, then, is given by the following equations,

$$f=\frac{N\times 10^5}{T}=\frac{(N-T)10^5}{T}+10^5=\left(1+\frac{N-T}{T}\right)10^5$$

where N=elapsed time in seconds indicated by the synchronous motor clock and T=time interval in seconds given by the time signals. The quantity $(N-T)$ represents the total gain or loss of time indicated by the synchronous-motor clock with respect to standard time. Letting $(N-T)=\pm D$, where plus is a gain in time and minus is a loss in time,

$$f=\left(1\pm\frac{D}{T}\right)10^5$$

If the time measurements are made for a time interval of 1 day or some integral multiple of a day,

$$f=\left(1\pm\frac{d}{86400}\right)10^5=10^5\pm 1.1574d,$$

where d=gain or loss in seconds per day.

The time interval to be chosen in this method of frequency determination is dependent on the constancy of the frequency standard itself and the accuracy with which it is desired to know the frequency. The length of interval must be short enough so that it is reasonably certain that the frequency was either constant during the interval or can be represented with the required accuracy by a linear variation. On the other hand, the interval must be long enough that the errors caused by the random variations in the determination of the time interval and the counting of the vibrations of the quartz plate are within the accuracy with which it is desired to determine the frequency.

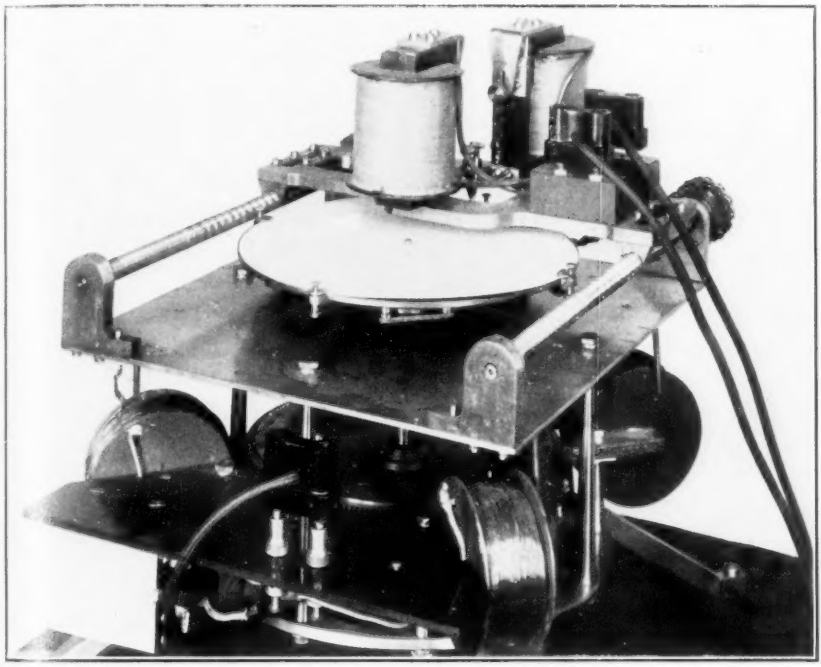

FIGURE 6.—*Electric chronograph driven by 100-cycle synchronous motor used for checking primary standard with NAA time signals.*

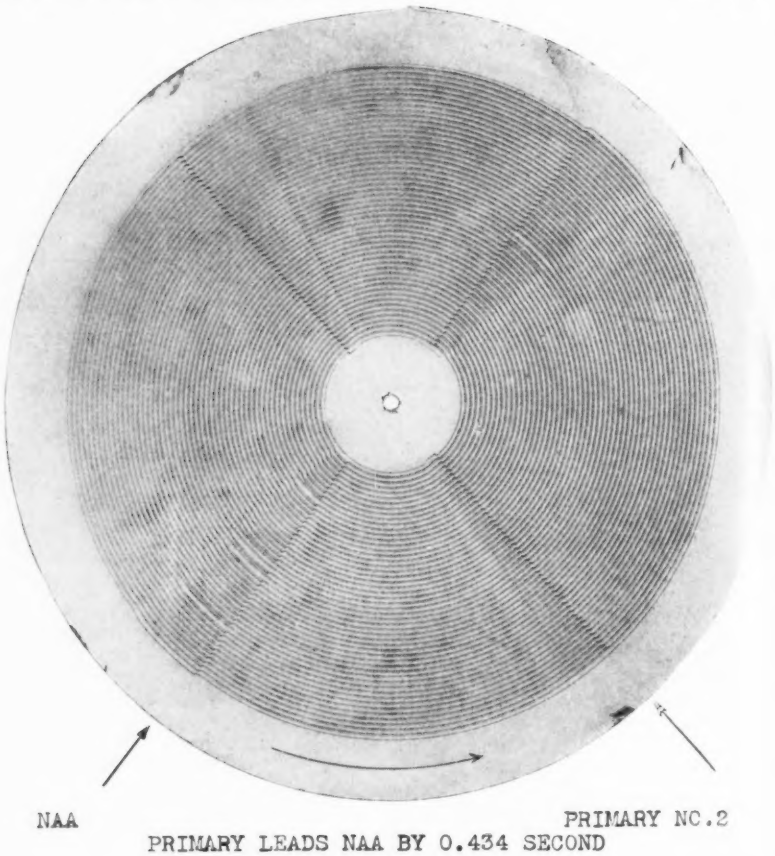

FIGURE 7.—*Sample record obtained with electric chronograph.*

A chronograph shown in figure 6 has a sheet of wax-coated paper mounted on a disk driven at a speed of 0.5 rps by a 100-cycle synchronous motor. Two styluses mounted on a carriage travel along a radius of the disk as it revolves and thus draw almost coincident spiral lines on the waxed paper. Second pulses from a cam-operated contact on the clock driven by the primary standard are impressed on one stylus, and radio time signals from the Naval Observatory on the other one, resulting in two sets of radial lines formed by the displacements in the spiral lines as shown in figure 7. The angle between each set of radial lines is read on a graduated circle and interpreted in seconds difference in time between the two time signals, as 180° on the paper record represents 1 second of time. This time difference may be accurately determined within a few thousandths of a second. The record on the disk of paper is made in a period of 1½ minutes. The value of the time difference as read is an average for that period of time.

In order to check the determination of the frequency of the primary standard, a number of additional measurements of time differences are taken each day. Both of the clocks operated from units of the primary standard are checked with the time signals from Naval Radio Station NAA to give a direct determination of the frequency. Both clocks are also checked against the Shortt clock at the Bureau. One clock is measured in terms of the other. With these various measurements available, simple computations will check the accuracy of the measurements and thereby minimize personal error.

Absolute measurements of frequency are made on only one of the piezo oscillators in each section. The beat indicators and the accompanying recorder make it possible to determine the frequency of the other units, as well as to provide a record of the operation of each standard in terms of the others. This record is the best information available as to the constancy of the standard controlling the synchronous-motor clock during the time interval over which the frequency is averaged. When first installed each beat indicator was connected to a telephone-message counter which added up the number of beats for the three pairs of piezo oscillators. A record of the beat counter readings, together with that of a seconds counter, was made photographically each 1,000 seconds. This method of recording required many moving parts which were quite difficult to keep in operation. Another disadvantage was the large amount of work required to develop, record, and compute the results.

The present beat-frequency recorder is an adaptation of a method of measuring low audio frequencies developed by the Bureau (3). The beat frequency, usually from 0.1 to 0.3 cycle per second, operates a polarized relay, T, figure 8. At each operation of relay T the condenser C_1 is discharged through the highly damped galvanometer G. Condenser C_2 and resistor R_2 act as a filter to smooth out the discharge current through G. Because of its long period of about 60 seconds the galvanometer takes a position depending on the average current through it, the current depending on the value of the beat frequency. The potential for charging condenser C_1 is obtained from the constant plate voltage of the primary standard.

The galvanometer used is a special microammeter made to record 6 successive measurements in different colors in approximately 17 minutes. One of these measurements is a calibration of the galvanometer made by a frequency of 0.2 cycle per second obtained by a

cam-operated contactor on the 1,000-cycle synchronous motor. This calibration is made frequently at 0.1 and 0.4 cycle per second. Figure 9 shows part of a recorder chart, with the central calibrating record and 5 beat-frequency records. Each small division of the scale represents a change in beat frequency of 0.004 c/s which represents, at the oscillator frequency of 100 kc/s, a variation of 4 parts in 100 million.

When it is desired to study the variations in frequency of any one of the piezo oscillators the frequency is multiplied to the 100th harmonic by a tuned multiplier circuit arrangement and beat against the 100th harmonic of another unit of the primary standard. This difference frequency is then recorded on a continuous recorder on which each small division represents a beat frequency change at 100 kc/s of 5 parts in 1 billion. With a record speed of 12 in./hr it is not difficult to note and time quite small variations in frequency difference. This recorder is of great assistance in locating and eliminating small frequency variations in the primary standard. It

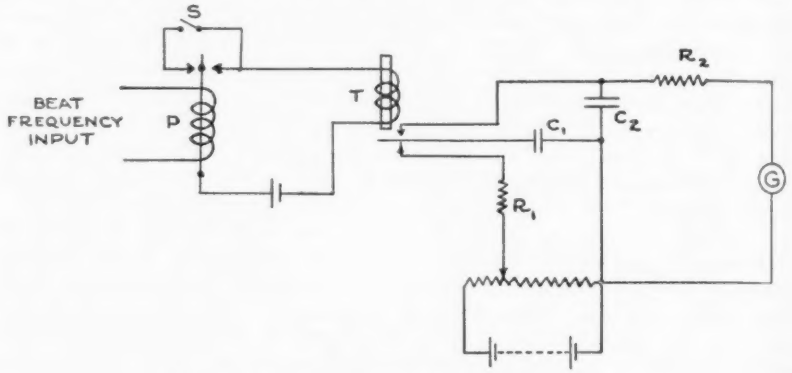

FIGURE 8.—*Graphic beat-frequency recorder.*

is expected that a description of the recorder and the accessory units will be published in the Journal of Research of the National Bureau of Standards under the title "Monitoring the standard frequency emissions", by Evan G. Lapham.

The frequency of the primary standard is calculated over a 10-day interval in terms of the uncorrected time signals and over a 6-day interval in terms of corrected time signals. The 10-day interval is required to give the desired accuracy as there is a maximum probable error in the time interval, as indicated by the uncorrected time signals, of 0.06 second which, if averaged over a 10-day period, reduces the maximum probable error in the frequency determination, due to this cause, to 7 parts per 100 million. This factor is considerably larger than the probable error caused by inaccuracies in the determination of the number of vibrations of the quartz plate in the time interval. The error in this measurement is not greater than ± 0.002 second or ± 200 cycles, which would cause an error in the frequency determination over a 10-day interval of only 0.5 part per 100 million. The maximum errors in terms of the corrected time signals over a 6-day interval are 4 parts per 100 million due to error in the time interval and 0.8 part per 100 million due to error in the determination of the vibrations of the quartz plate.

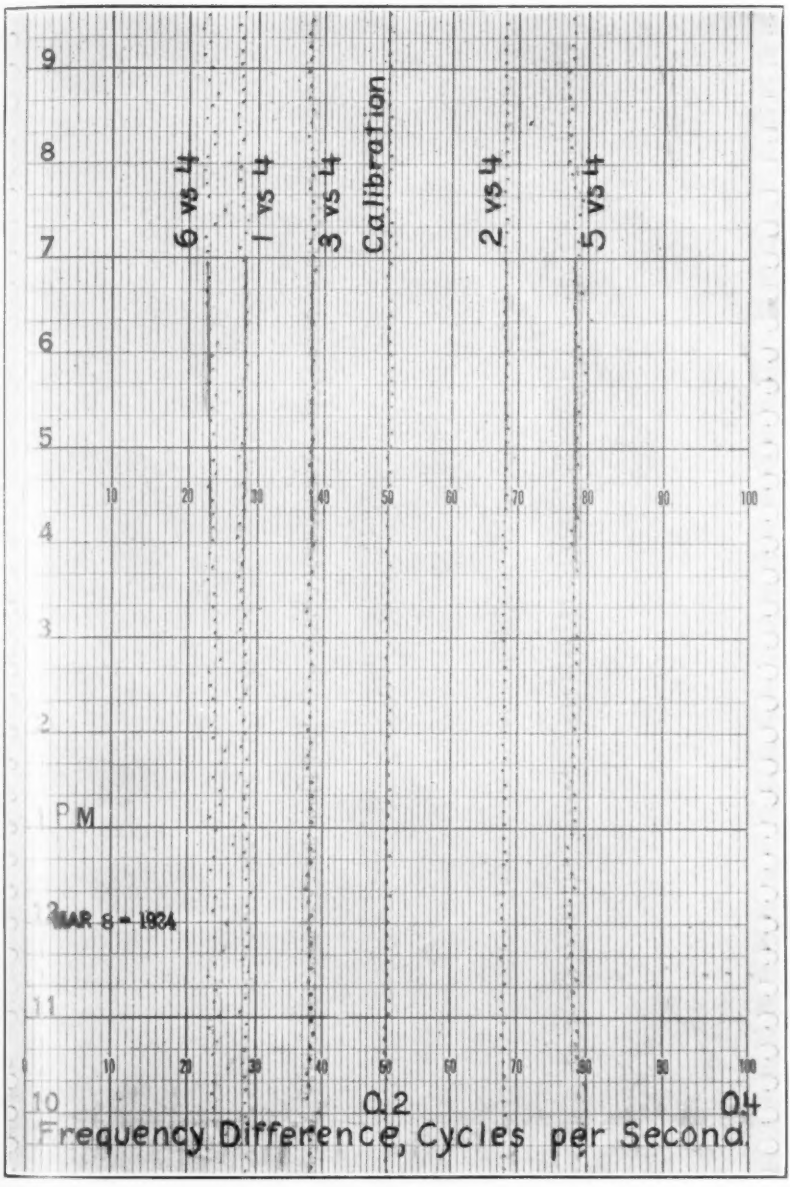

FIGURE 9.—*Sample record obtained with graphic beat-frequency recorder.*

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The frequencies of the units of the primary standard are very seldom constant. The fact that a frequency drift exists and that the value of the frequency, derived from a comparison with the standard time interval is the average for the time interval and occurs at the midpoint of the interval, necessitates extrapolating the data obtained from the uncorrected time signals ahead for 5 days. To facilitate this extrapolation the frequency is plotted on cross-section paper so that the average rate of change of frequency can be readily determined. The final corrected values are plotted on the same sheet with the values obtained from the uncorrected time signals. Using this value of the frequency and the beat indicator record, daily values are computed for the frequencies of each of the other units of the primary standard.

IV. USES OF THE PRIMARY FREQUENCY STANDARD

The advantage to any radio laboratory of having available a frequency standard of great accuracy, giving submultiple frequencies as described above, is apparent. The application of such a device to many electrical and scientific investigations will likewise be readily apparent. A number of cases where these frequencies are being advantageously used at the Bureau are given below. However, one of the most important applications of the national primary frequency standard is its dissemination for use by the general public for checking frequency standards maintained by radio and electrical instrument manufacturers and testing laboratories, radio transmitting stations, university laboratories and others desirous of accurate frequency values. Dissemination of the frequency standard is accomplished by two methods, first by radio transmission and second by direct wire connection to one of the primary frequency standards.

The general method of providing standard frequencies by radio transmissions has been in use by the Bureau for eleven years. During the last four years a transmission on 5,000 kc/s (4) was available four hours a week. These transmissions were useful throughout most of the United States. As these transmissions are held accurately upon frequency by comparison with the primary standard, anyone using the transmissions in effect has direct access to the primary standard.

The method of dissemination of frequencies from the primary standard by means of a direct wire connection to one of its units is limited to a small area in the vicinity of Washington because of the expense of a long wire line. Such a connection has been used for over two years by one laboratory near Washington. A frequency of 10 kc/s from one of the primary standards is impressed upon a telephone line leased by the measurement laboratory.

A frequency of 1000 cycles per second from the primary standard is used in the Bureau laboratories in the measurement of absolute value of resistance, and in the calibration of inductors and mica condensers. This frequency is also used in calibration of the tuned reeds for the reed indicators used in directing airplanes on their courses.

A frequency of 100 cycles per second from the primary standard is used in measurements on air condensers and in the calibration of audio-frequency standards. This frequency is amplified to drive the synchronous motor of the time signal chronograph. A frequency of 60 cycles per second provided by section II of the primary standard is used to calibrate the reeds in reed indicators for airplanes and as a tim-

ing frequency for velocity meter tests. This frequency is also used to control the frequency of a 60-cycle generator used in the testing of electrical instruments. A frequency of 10 cycles per second is useful in calibrating audio-frequency standards.

The one-second contactor is used for intercomparing time rates of clocks driven by the primary standard and various time standards, and also as a source of constant seconds pulses for counting numbers of α-particles emitted by a radium sample. The frequency of 0.2 cycle per second serves to calibrate the beat recorder and also as a source of constant signal pulses for counting α-particles emitted by a radium sample.

V. PERFORMANCE AND DISCUSSION

Since the installation of section I of the primary standard, many changes have been made in the individual units themselves, in the wiring, and in the instruments and methods of calculating the frequency, all of which have added to the accuracy with which the frequency is known. Certain limitations are recognized as present which may perhaps be removed later. There is considerable evidence that the units of section I are subject to similar variations in frequency. This may be due to the similarity in their construction or to stray coupling between the standards. The fact that the output amplifiers use triodes which have considerable grid-plate capacity, and also that the units have a common power supply make it seem all the more likely that a small amount of coupling might be present. Frequency checks with one or more units operated on a separate power supply would show that one or the other section had changed. The advantage of two independent sections is therefore evident.

The improvements in the equipment since its installation have resulted in a considerable extension of the periods of continuous operation of the synchronous-motor clock. The longest period of continuous operation of the synchronous-motor clock in section I is a little over seven months. The synchronous-motor clock in section II operated without stoppage for about the same length of time. The average period of continuous operation for the two synchronous-motor clocks for the last two years is over four months. Accidental stoppage and failure of the 1000-cycle output due to aging of the amplifier tubes account for most of the discontinuity in operation. Contrary to expectations, only in a very few instances have the stoppages of the synchronous-motor clocks been traceable to the submultiple generators. As far as the determination of the frequencies of the standards is concerned, the continuity of operation of the synchronous-motor clocks is quite satisfactory. Interruptions in the operation of the clocks, however, would interfere greatly with their usefulness as time standards.

Frequency variations in piezo oscillators may be of two main types: (a) long-time variations or those which would not be noted in intervals of a few hours or a day, but may be noticed over a period of several weeks or months; (b) short-time variations or those which may be measured and may occur irregularly, perhaps instantaneously, or in intervals of a few minutes or less. The frequency variations tolerable in a unit of a primary frequency standard are extremely small in comparison with variations which might be of little consequence in other frequency standards.

An oscillator which is either constant in frequency or displays only a long-time variation is most useful as a frequency standard, as its change in frequency can be predicted and corrections made for the slight drift or change in value. This type of oscillator is convenient for driving a timekeeping mechanism and may ultimately compete with the most exact pendulum clocks used in observatories (5). The performance of this type of timekeeper is of great interest because of its entirely different principle of operation, being unaffected by some factors to which all pendulum clocks are subject.

An oscillator subject to short-time variations is not reliable for instantaneous measurements to better than the order of the maximum short-time variations observed. The measurements could conceivably be made at the time when one of the maximum variations had taken place and would accordingly be in error by that amount.

Piezo oscillators of different designs usually display both of the types of frequency variation mentioned above in different amounts. Each piezo oscillator therefore must be considered individually and efforts made to determine its characteristics and reliability in maintaining its frequency. Where instantaneous measurements of the greatest accuracy are desired, an oscillator showing minimum short-time variations should be employed.

Measurements on the several units of the primary standard show maximum short-time frequency variations of different amounts ranging from less than 1 to several parts in 100 million. The oscillators do not show the same characteristics continually but may change with time. One that has maintained a fairly constant frequency for a long period may change and become unreliable for the highest precision. The opposite case may also occur. It is therefore desirable to periodically check up on the short-time frequency changes of each oscillator. Knowing the details of construction of the several oscillators, it is often possible to account for part of the peculiar behavior observed in some of the standards. None of the instruments used with the primary standard have, however, afforded information by which to predict that an oscillator was about to alter its rate of change of frequency.

It is of interest to know what accuracy can be expected from a group of piezo oscillators that operate continuously, under controlled conditions, as described above. For this purpose the curves of figure 10 have been prepared showing the frequency change with time for the two units of the primary standard which drive the synchronous-motor clocks. To obtain these curves, the daily corrected frequencies for the 1st and the 15th day of each month were plotted. The upper curve covering a period of 21 months shows the frequency variations of no. 12 of section I. The breaks in the curve indicate times when frequency adjustments were made.

The upper curve is representative of the standards in section I. It should be noted that the frequency of these units may drift either higher or lower at a rate of ± 3 parts in 10 million per month. The average frequency over a period of a year or more, however, increases in value. Although it is not apparent in the curve shown, the average rate of change of frequency would be of the order of 1 part in 1 million per year. The maximum deviation from the average is approximately ± 1 part in 1 million.

The lower curve of figure 10 shows the frequency variations of no. 26 of section II, for a period of 15 months. This is the experimental

unit previously mentioned which is not equipped with pressure control. If the pressure had been maintained more uniform, a more regular curve would probably have been obtained. During this period the frequency has been subject to a fairly gradual increase of approximately 1 part in 10 million per month. Although there have been periods when the frequency has drifted at a somewhat greater rate, these periods have been of comparatively short duration, so that the maximum deviation from the average is of the order of ± 2 parts in 10 million. Owing to the shortness of the period of operation, however, any statement as to its over-all frequency drift is highly speculative.

It is quite an advantage to have section II of the primary standard operated by separate power supply units and with circuit arrangements differing from those of section I. Whenever variations in frequency take place, it is unlikely that all six units of the primary standard would vary alike. If the voltage-control system of one sec-

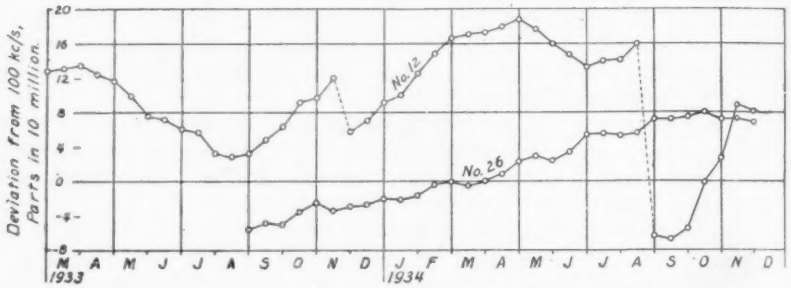

FIGURE 10.—*Frequency deviation curves for primary nos. 12 and 26.*

tion fails to operate properly, the other section continues to function properly and serves as a basis for frequency measurements.

The authors wish to express their appreciation of the contributions to the satisfactory operation of the primary standard made by the following persons: C. G. McIlwraith and A. H. Hodge for the installation of section I and development of measuring equipment; R. B. Wright for the experimental developments leading to the construction of primary 26 of section II.

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SPECIFIC VOLUME, COMPRESSIBILITY, AND VOLUME THERMAL EXPANSIVITY OF RUBBER-SULPHUR COMPOUNDS

By Arnold H. Scott

ABSTRACT

The specific volume, compressibility, and volume thermal expansivity of rubber-sulphur specimens containing from 3 to 31 percent of sulphur have been determined at temperatures from 10 to 85° C for pressures up to 800 bars (790 atmospheres). The specific volume was determined at atmospheric pressure by means of a pycnometer. The specific volumes at the other pressures were obtained from measurements of the lengths of rods of the materials as the pressures were applied. The compressibilities and volume thermal expansivities were computed from these results. The effect of pressure on the specific volume was found to be a function of the sulphur content and the temperature. The specific volume may be computed from the equation $V=V_a(1+\alpha P+\beta P^2)$, where V_a is the specific volume at atmospheric pressure, P is the pressure, and α and β are coefficients. For temperatures between 10 and 85° C and for specimens containing up to X percent of sulphur where X is given by the equation $X=(T+30)/3.7$ and T is the temperature in degrees centigrade, V_a, α and β have the following values:

$$V_a=1.1015+0.00073(T-25)-X[0.00932+0.000007(T-25)]$$
$$\alpha=-10^{-6}\{53.7+0.258(T-25)-X[1.35-0.00217(T-25)]\}$$
$$\beta=10^{-9}[12.0+0.083(T-25)-0.34\ X]$$

For temperatures between 10 and 85° C and for specimens containing between X and 32 percent of sulphur where X is given by the equation $X=(T+42)/3.7$, V_a, α, and β have the following values:

$$V_a=1.0450+0.00038(T-25)-X[0.00601+0.0000064(T-25)]$$
$$\alpha=-10^{-6}\{35.6+0.326(T-25)-X[0.352+0.00672(T-25)]\}$$
$$\beta=10^{-9}[9.1+0.0683(T-25)-0.221\ X]$$

The volume thermal expansivity for the specimen containing 3 percent of sulphur decreased from 6.7×10^{-4} at 1 bar to 5.0×10^{-4} at 800 bars, while for the specimen containing 31 percent of sulphur, it decreased from 2.2×10^{-4} to 1.7×10^{-4}. The compressibility varied from a maximum of 66×10^{-6} to a minimum of 21×10^{-6}.

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I. INTRODUCTION

The specific volume, compressibility, and volume thermal expansivity of rubber-sulphur compounds [1] containing from 3 to 31 percent of sulphur have been measured at pressures from 1 to 800 bars and at temperatures from 10 to 85° C. The results are expressed in tables, curves, and equations so that, within the indicated limits, the value of any of the above properties can be readily determined for a compound of specified composition under specified conditions of temperature and pressure.

Specific volume, which is the reciprocal of the density, has been used throughout this paper because this term is the one which is used directly in the definitions of compressibility and volume thermal expansivity. The work of Kimura and Namikawa [2] indicated that the specific volume of rubber-sulphur compounds at atmospheric pressure was a linear function of the temperature. The use of the specific volume thus leads to equations which are simpler than if the density is used.

II. METHOD OF DETERMINING SPECIFIC VOLUME

The specific volume of a material at atmospheric pressure was determined by means of a pycnometer. The specific volumes at other pressures were obtained from measurements of the changes in length of a rod of the same material when subjected to different pressures.

The procedure used in the determination of the change in length was to make measurements at successive pressures through a pressure cycle at a fixed temperature. The pressure was increased in steps of 200 bars [3] up to 800 bars, allowing time for temperature equilibrium in each case. The same measurements were then made as the pressure was decreased by steps of 200 bars back to atmospheric pressure. A group of measurements obtained when carrying the specimen through this pressure cycle is called a set of data. A repetition of this at the same or another temperature is considered another set.

The specific volume at a given pressure was computed from the change in length of the rod when the pressure was applied. Assuming the material of the rod to be isotropic, then the volume change with pressure is proportional to the cube of the length.[4] Let

L_a = the length of the rod at atmospheric pressure (approximately 1 bar)

ΔL = the change in length of the rod when the pressure has been been increased to any pressure, say P bars

V_a = the specific volume at atmospheric pressure (approximately 1 bar)

V = the specific volume at P bars

[1] This paper is one of a series dealing with the properties of rubber-sulphur compounds. See BS Tech. Pap. **19**, 669 (1925) T299; BS Sci. Pap. **22**, 383 (1927) S560; and BS J. Research **11**, 173 (1933) RP585. A need for information regarding the compressibility of rubber arose in connection with the determination of the effect of pressure on the electrical properties of rubber where a knowledge of the change in dimensions was required.

[2] J. Soc. Chem. Ind., Japan, **32** (supplemental binding) 196B (1929).

[3] The bar is defined as 10^6 dynes/cm^2 and is equal to 0.987 normal atmosphere.

[4] The assumption that the samples were isotropic was based on the method of preparation and is confirmed by recent photoelastic studies on vulcanized rubber by Thibodeau and McPherson, J. Research NBS **13**, 887 (1934) RP751.

Then

$$\frac{V}{V_a}=\frac{(L_a-\Delta L)^3}{L_a^3}$$

or

$$V=V_a[1-3\Delta L/L_a+3(\Delta L/L_a)^2-(\Delta L/L_a)^3] \quad (1)$$

The last term was found to be negligible for pressures used in this investigation.

The specific volume was found to vary with the pressure in a manner described by the equation:

$$V=V_o(1+\alpha P+\beta P^2) \quad (2)$$

where P is the pressure in bars, V_o is the specific volume at zero pressure, and α and β are characteristic coefficients which depend on the temperature and the composition of the rubber-sulphur compounds.[5] V_o is equal to V_a within experimental error and therefore V_a is used in the above equation in place of V_o. The approximate values of the coefficients may be obtained from measurements of change in length by selecting two observations from a set, preferably the change in length at maximum pressure and the change in length at about one-half maximum pressure. Let

V_1=specific volume at P_1 bars

V_2=specific volume at P_2 bars (about 2 times P_1)

ΔL_1=change in length from 1 to P_1 bars

ΔL_2=change in length from 1 to P_2 bars

Then from eq 2

$$V_1=V_a(1+\alpha P_1+\beta P_1^2)$$

$$V_2=V_a(1+\alpha P_2+\beta P_2^2)$$

and from eq 1, neglecting the cubic term

$$V_1=V_a[1-3\Delta L_1/L_a+3(\Delta L_1/L_a)^2]$$

$$V_2=V_a[1-3\Delta L_2/L_a+3(\Delta L_2/L_a)^2]$$

Solving these equations we find

$$\alpha=\frac{3}{P_2-P_1}\left\{\frac{P_1}{P_2}\left[\frac{\Delta L_2}{L_a}-\left(\frac{\Delta L_2}{L_a}\right)^2\right]-\frac{P_2}{P_1}\left[\frac{\Delta L_1}{L_a}-\left(\frac{\Delta L_1}{L_a}\right)^2\right]\right\} \quad (3)$$

$$\beta=-\frac{3}{P_2-P_1}\left\{\frac{1}{P_2}\left[\frac{\Delta L_2}{L_a}-\left(\frac{\Delta L_2}{L_a}\right)^2\right]-\frac{1}{P_1}\left[\frac{\Delta L_1}{L_a}-\left(\frac{\Delta L_1}{L_a}\right)^2\right]\right\} \quad (4)$$

The values of α and β obtained from these equations for a given specimen were used in eq 2 and the resulting equation was then checked against all the values of the specific volume of this specimen,

[5] The cubic term in the equation which Adams and Gibson found to describe the change in volume of rubber with pressure (see J. Wash. Acad. Sci. **20**, 213 (1930)) was unnecessary over the short pressure range used in the present investigation.

determined from its change in length. A slight revision in the values of the coefficients was necessary in a few cases in order to make the equation more closely represent the complete data of a set.

Compressibility, σ, is defined as the decrease in volume per unit volume for a unit increase in pressure at constant temperature. That is

$$\sigma = -\frac{1}{V_0}\frac{dV}{dP} \tag{5}$$

Thus by differentiating eq 2 we obtain

$$\sigma = -\alpha - 2\beta P \tag{6}$$

The compressibility may be determined by simply giving α and β the values computed from the equations above.

The volume thermal expansivity is defined as the increase in volume per unit volume for an increase of 1° of temperature at constant pressure. Since the volume thermal expansivity was determined only for those temperature ranges in which the specific volume was a linear function of the temperature, the expansivity was computed from the slope of the line obtained by plotting the specific volume against the temperature.

III. DESCRIPTION OF APPARATUS

The specific volume measurements at atmospheric pressure were made with the pycnometer described by Ashton, Houston, and Saylor.[6] A sketch of this pycnometer is shown in figure 1. The pycnometer was made of glass and had a capacity of about 25 cm^3.

The specific volume was determined from the weight of the specimen and the weight of the pycnometer containing first the confining liquid and then the confining liquid and specimen. Water was used as the confining liquid. Investigation showed that the absorption of water by the rubber-sulphur compounds had a negligible effect on the results. The height of the confining liquid in the capillary tube was adjusted after the pycnometer and contents had been kept in a constant temperature bath for 30 minutes to insure uniform temperature. The bath was controlled to 0.1° C and weighings were made to 1 mg. After adjustment at the desired temperature the pycnometer and its contents were brought to a temperature near that of the room before it was weighed. This was done by placing it in a water bath at room temperature for a few minutes.

No grease was used on the ground-glass joints of the pycnometer and therefore slow evaporation of the water took place, making the weight a function of the time which elapsed after the liquid was adjusted in the capillary tube. However, a strict time schedule was employed so that measurements could be repeated to 5 mg or less, which resulted in an error of less than 0.1 percent in the determination of specific volume.

The specific volume, in most cases, was also determined at 25° C on the entire specimen used in the pressure measurements. This was determined by weighing the specimen in air and in water.

[6] BS J. Research **11**, 243 (1933) RP587. This pycnometer is a modification of the one used by Richards, Hall, and Mair. J. Am. Chem. Soc. **50**, 3308 (1928).

The change in length of a specimen with pressure was determined with the apparatus shown diagrammatically in figure 2. The specimen, which was inclosed in the pressure chamber, was suspended at its upper end. The glass scale which was fastened to the lower end extended between the two glass pressure windows. The scale was illuminated through one window and viewed by means of a microscope through another window.

The pressure chamber consisted of a steel pipe known commercially as "double extra heavy", having a heavy fitting on each end. The upper fitting had two openings, one of which allowed the insertion of the specimen and the other the connection to the pressure pump. The lower fitting had two openings which were opposite each other and into which the steel plugs carrying the pressure windows were screwed. The fittings were equipped with lead rings which could be firmly pressed into the threads by means of set screws, thus producing tight joints.

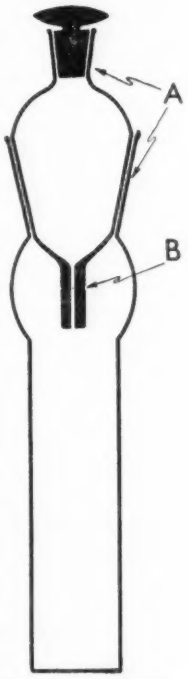

FIGURE 1.—*Apparatus for specific volume determinations at atmospheric pressure.*

A, ground glass joints; B, volume adjustment mark.

The pressure chamber was hung from a bracket on a brick wall. This bracket had a flat horizontal iron shelf about 9 mm thick from which was cut a U-shaped section of such size that the plug in the upper end of the pressure chamber would just fit into it. Thus the weight of the chamber was supported by the shoulder of the plug resting on the shelf. The microscope used to view the scale was mounted on another bracket which was also fastened to the brick wall. Since a set of measurements was made at constant temperature, the distance between the brackets was assumed to remain fixed during this set of observations. Any changes in dimensions of the pressure chamber due to pressure changes had no effect on the measurement of the length of the rubber rod. A prism which was mounted on the bracket with the microscope and which is not shown in the figure was used so that the microscope could be mounted at right angles to the line through the pressure windows. This allowed the operator to make observations while facing the wall and kept the face of the observer out of danger in case the pressure window failed. The lower end of the pressure chamber was held against two guides by spring clips not shown in the figure.

The rods of soft rubber were fastened to the plug at the upper end by cementing them with a commercial thermoprene cement. The rods of hard rubber were screwed to the plug. A machine screw about 6 mm long was fastened to the end of the plug. A hole was drilled and tapped in the end of the rod and this was then screwed onto the machine screw. Some of the semihard rubbers were both screwed and cemented to the plug.

An aluminum rod 17 cm long was fastened to the lower end of the rubber rod by the same method used to fasten the rod to the plug.

This metal rod was necessary because the density of some of the soft rubbers was less than that of water and a weight was needed to counteract the tendency of the rod to float. An aluminum rod having a weight sufficient to supplement the weight of the least dense specimen was used on all specimens. The excess weight of the metal rod for the more dense specimens did not affect the measurements appreciably since this excess weight was small and the stiffness of the rubber specimens increased with the density.

The engraved glass scale, which was 3 cm long, was fastened to the lower end of the aluminum rod by means of a frame and clip. The

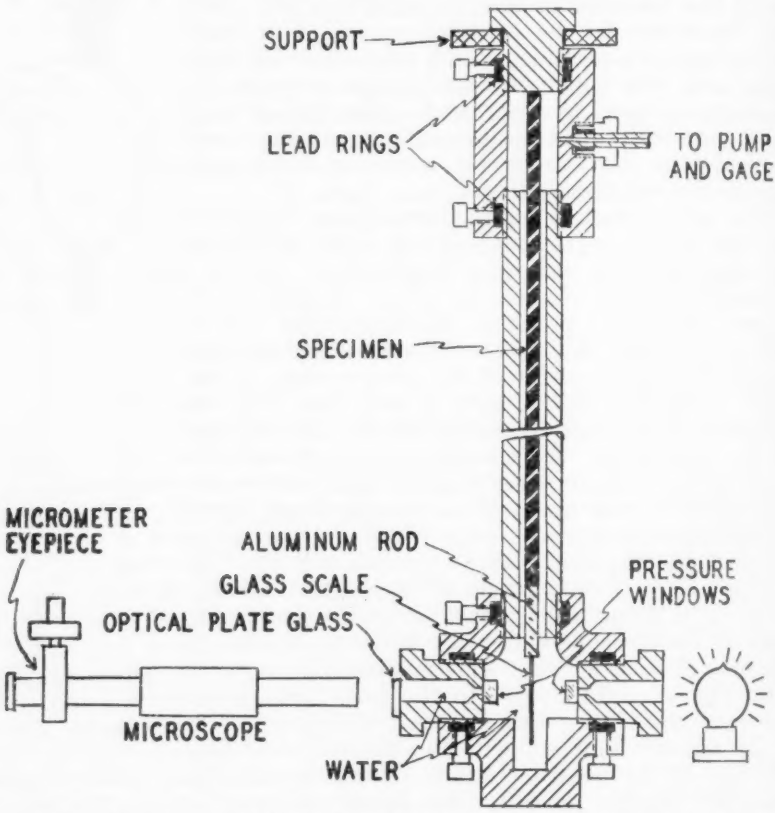

FIGURE 2.—*Apparatus used in the determination of the change in length of a cylindrical specimen with pressure.*

clip kept the end of the scale pressed firmly against the end of the rod. The changes in position of the scale were due to changes in length of both the rubber and aluminum rods. The small corrections for the change in length of the aluminum rod were computed from the values of compressibility given in the International Critical Tables.[7]

The scale was ruled with a diamond point on the platinized surface of a piece of optical plate glass. Since the scale was illuminated from behind, the lines appeared bright on a dark field. The lines

[7] International Critical Tables 3, 46.

were ruled on a precision dividing engine at 0.5 mm intervals and were designated by engraved numbers. This scale was used to calibrate the micrometer eyepiece of the microscope. One millimeter was equivalent to 346 divisions on the micrometer eyepiece.

The motion of the scale and hence the change in length of the specimen was measured by means of the micrometer eyepiece of the microscope. Three lines of the scale were visible in the field of the microscope. The crosshair was set on the line nearest the center of the field and the change in length of the rod was determined from the reading of the micrometer eyepiece and the number of the line on the scale. The approximate total length was obtained from the same readings since the length from the upper support to the center of the field of view of the microscope was measured directly with a meter stick. This distance was determined to 0.5 mm and the length of the specimen was obtained by subtracting from this the length of the metal rod and scale above the center of the field of view.

The glass pressure windows were supported by hardened plugs of stainless steel each of which had a hole bored through its center. The diameter of this hole was 6 mm at the inner end of the plug where it was covered by the window. This was increased to about 15 mm at the other end of the plug. The seal between the glass and the steel plug was made by grinding the two surfaces optically flat, allowing the pressure of the liquid to force the glass firmly against the steel. This effectively sealed the joint since the bearing surface was appreciably less than the surface exposed to the pressure. It was necessary to put a thin film of canada balsam under the window to hold it in place until the pressure was applied.

Water was placed in the opening of the plug through which the scale was viewed. This was done to minimize the distortion of the field of view in the microscope due to warping of the glass window under pressure. The water had an index of refraction which was much nearer that of the glass than was that of air. Therefore the refraction at the water-glass surface was much less than that at an air-glass surface, making the effect of the warping of the window on the field of view less when the window was surrounded with water. In fact, surrounding the window with water reduced the change in refraction due to warping of the window to such an extent that it was found to be negligible for the pressures used in this investigation. A piece of optical plate glass was cemented over the opening at the outer end of the plug to keep the water in place and to give a refracting surface which was not affected by the pressure.

The pressure chamber was mounted in a constant temperature air bath provided with numerous baffles and valves so that the air flow could be readily changed and the same temperature could be maintained throughout the pressure chamber. The temperature of the chamber was measured by 3 thermocouples soldered to it, one near the top, one near the middle, and one at the bottom. It was possible, with close watching, to keep the temperature of the chamber as indicated by the 3 thermocouples within 0.1° C of the desired temperature.

The pressure was measured by a resistance gage of the type described by Gibson.[8] A diagram of this gage is shown in figure 3. The manganin resistance coil was placed in a steel chamber which

[8] J. Am. Chem. Soc. **56**, 4 (1934).

was connected to the pressure chamber by small-bore steel tubing. The leads to the coil were made of piano wire and were brought out through a packing of limestone, rubber, and talc. The limestone and talc were machined to a tight fit. The rubber was made oversize and had to be squeezed into place. The whole packing was forced into place with a pressure of several thousand pounds. To do this a piece of steel was made to hold the plug and a plunger was made to place over the packing. This was then placed in a press and the packing forced into place. The packing was kept in place by the friction against the sidewalls. Pressure applied to the gage chamber

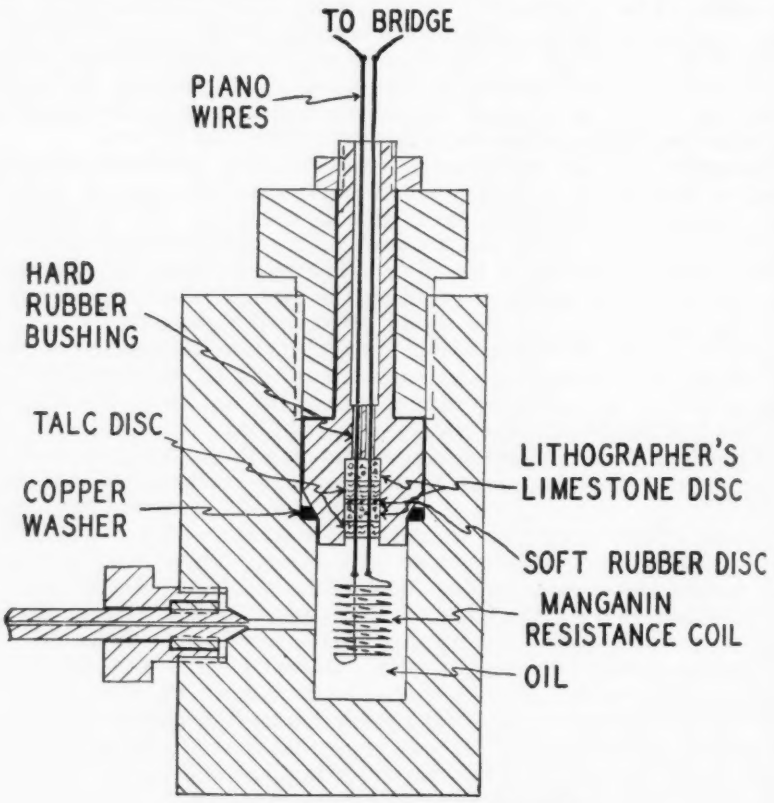

FIGURE 3.—*Resistance pressure gage.*

forced the packing closer into the plug opening and more effectively sealed it. There was no tendency for the lead wires to creep with this method of packing.

The coil was made of no. 38 manganin wire and had a resistance of about 101 ohms. It was wound in a single layer on a light cylinder of very thin paper. The form was preserved by covering the coil with a thin coat of shellac and baking it in an oven for several hours. The coil was seasoned under a pressure of 8,000 bars. A light insulating oil was used as the pressure medium surrounding the coil. Since water was used in the rest of the pressure apparatus it was necessary to put a U-tube filled with mercury in the pressure line leading to the gage.

The resistance of the coil was measured with a Wheatstone bridge capable of measuring resistance to 0.0001 ohm. The gage and bridge were placed in a carefully controlled temperature bath of oil. A change in pressure of 1 bar changed the resistance of the coil about 0.0002 ohm so that a change in pressure of ½ bar could be readily measured. Twenty to thirty minutes were allowed after the pressure had been applied for conditions to reach equilibrium before measurements were made, since a change in pressure affected the temperature equilibrium.

IV. PREPARATION OF SPECIMENS

The specimens were made from purified rubber and sulphur and were molded in the form of rods about 1 cm in diameter and 85 cm long. The sulphur content of the different specimens ranged from 3 to 31 percent with no interval greater than 6 percent. The specimens were analyzed for sulphur content after they were vulcanized. The sulphur content was determined by burning a weighed sample of the rubber in a bomb with oxygen, absorbing the products of combustion in an alkaline solution, and determining the sulphur as barium sulphate as described by Mease.[9]

The purification of the rubber and the mixing of the rubber and sulphur were carried out as described in a former paper.[10] The specimens were vulcanized in a mold in steam at 148° C for about 20 hours. This relatively long time of vulcanizing was for the purpose of bringing practically all the sulphur into combination with the rubber.

The specimens were carefully examined for porosity. The density of the rod as a whole was determined by weighing it in air and in water. Any rod which had a density appreciably less than the value to be expected from previous experience with specimens of similar composition was discarded.

V. RESULTS

The determinations of the specific volume are divided into two groups, those which were obtained with the pycnometer at atmospheric pressure and those which were obtained from the measurement of the change of length with pressure. The determinations at atmospheric pressure with the pycnometer were much more accurate and reproducible than those made under pressure. These results and the compressibility are presented in the form of curves. The values of the volume thermal expansivity are given in a table.

1. SPECIFIC VOLUMES AT ATMOSPHERIC PRESSURE

The values of the specific volumes at atmospheric pressure for various percentages of sulphur are shown in figure 4 where they are plotted against the temperature. The specific volume of the specimen containing 6 percent of sulphur was measured directly at 25° C only. The values at the other temperatures, as indicated by open circles, were obtained by interpolation when the specific volumes were plotted against the percentage of sulphur for each temperature.

[9] J. Research NBS **13**, 617 (1934) RP731.
[10] Scott, McPherson, and Curtis. BS J. Research **11**, 173 (1933) RP585.

The relation between the specific volume and temperature is linear for specimens containing up to 16.5 percent of sulphur. The relationship for specimens containing higher percentages of sulphur is

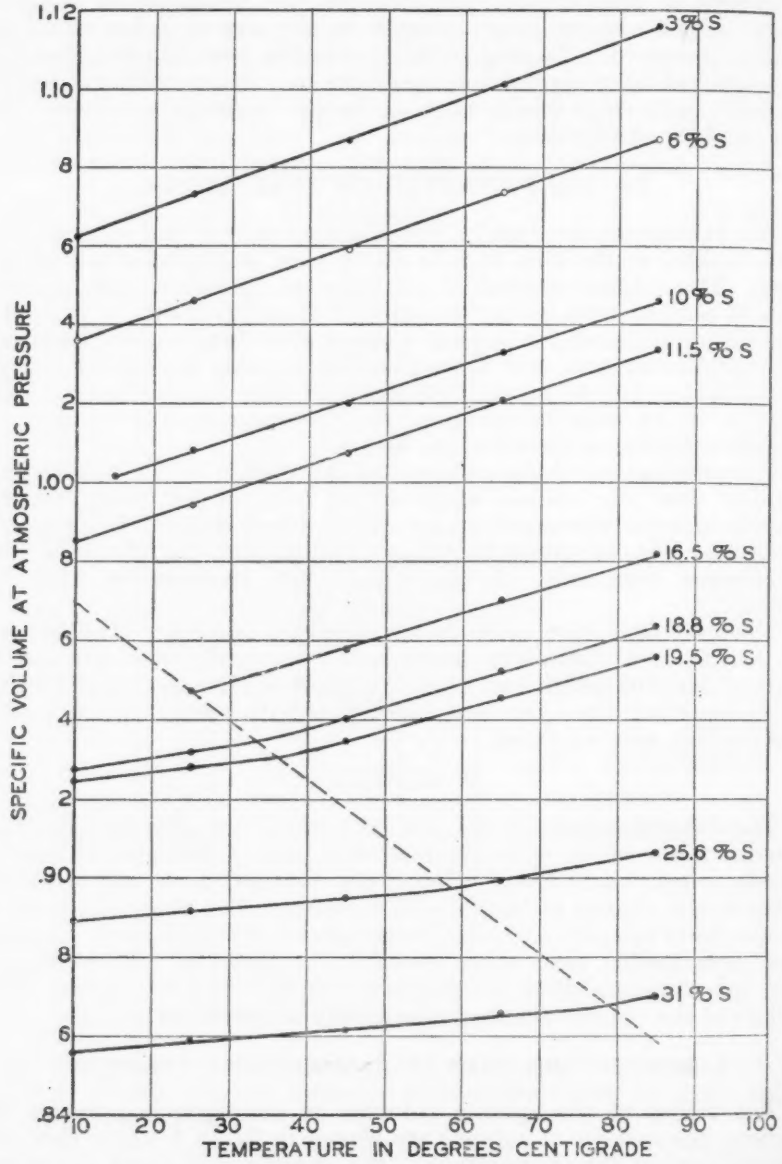

FIGURE 4.—*Effect of temperature on the specific volume of rubber-sulphur compounds at atmospheric pressure.*

The values represented by open circles were obtained by interpolation when the specific volumes were plotted against the percentage of combined sulphur.

such that the curve is linear in two sections but the slopes of the two sections differ. The slope for the higher temperatures is about that for the specimens containing 16.5 percent of sulphur or less

Thus it is seen that if a line is drawn from approximately 0.97 at 10° C to approximately 0.86 at 85° C, as shown in the figure, the curves or parts of curves above this line all have about the same slope and the parts of curves below this line have about the same slope, but the slope of the latter is different from the slope of the curves above the line. This was also observed and reported by Kimura and Namikawa.[11] Bekkedahl [12] reported changes in slope of the curves for 2, 4, and 6 percent of sulphur at low temperatures.

2. SPECIFIC VOLUMES AT HIGH PRESSURES

The specific volumes at 200, 400, 600, and 800 bars pressure are shown plotted against the temperature in figures 5 to 8. These values were obtained from measurements of the changes in length of the specimens under pressure. These curves have the same general form as those at atmospheric pressure, being merely shifted in position. The spread of the points increased with the pressure, in some cases making the determination of the slope more difficult for the higher pressures. However, it was readily determined that the slope of all the curves decreased with pressure, which means that the volume expansivity decreased with pressure. Each curve was shifted to lower values of specific volume as the pressure was increased. The amount of the shift was greater for the low percentages of sulphur than for the high percentages. The line marking the boundary between the curves or parts of curves having the smaller slopes and those having the larger slopes shifted to lower values of specific volume as the pressure was increased. At 800 bars the approximate position of this boundary line was from 0.94 at 10° C to 0.84 at 85° C.

The manner in which the specific volume decreased with pressure at various temperatures is better shown in figures 9, 10, and 11. The curves in these figures were obtained from the data shown in the previous figure. The percentage change of the specific volume with a pressure change of 800 bars increased with the temperature and decreased with the percentage of sulphur. For instance, the specific volume of a specimen containing 3 percent of sulphur is decreased about 3.3 percent at 25° C by a pressure of 800 bars, and about 4.2 percent at 85° C. The corresponding decreases for the specimen containing 31 percent of sulphur are 1.8 and 2.4 percent.

The reproducibility of the results can be seen from the spread of the points in figures 4 to 8, or from the curves in figures 9, 10, and 11. Where two curves are drawn they represent the maximum and minimum values. When more than one set of measurements was made and only one line has been drawn, the values were so close together as to be indistinguishable on the scale used. The number of sets of measurements made for each composition at each temperature is shown in table 1.

[11] J. Soc. Chem. Ind., Japan, **32** (supplemental binding), 196B (1929).
[12] J. Research NBS **13**, 411 (1934) RP717.

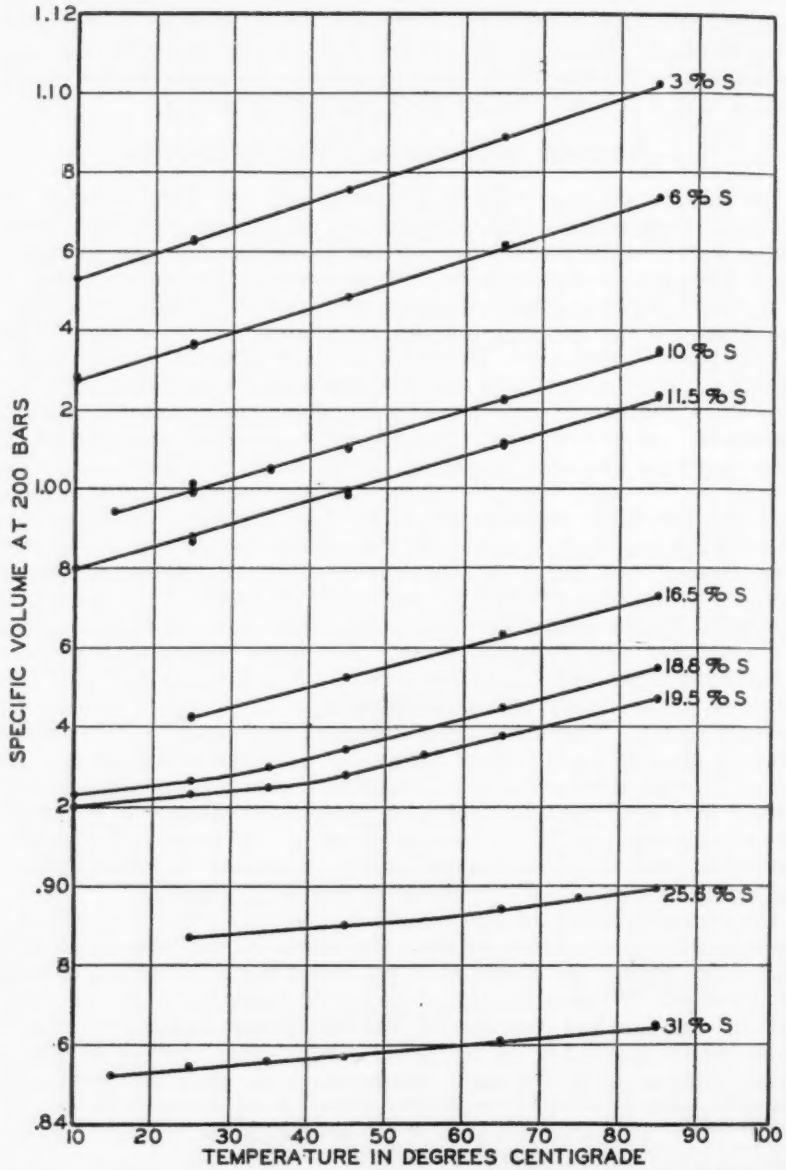

FIGURE 5.—*Effect of temperature on the specific volume of rubber-sulphur compounds at 200 bars pressure.*

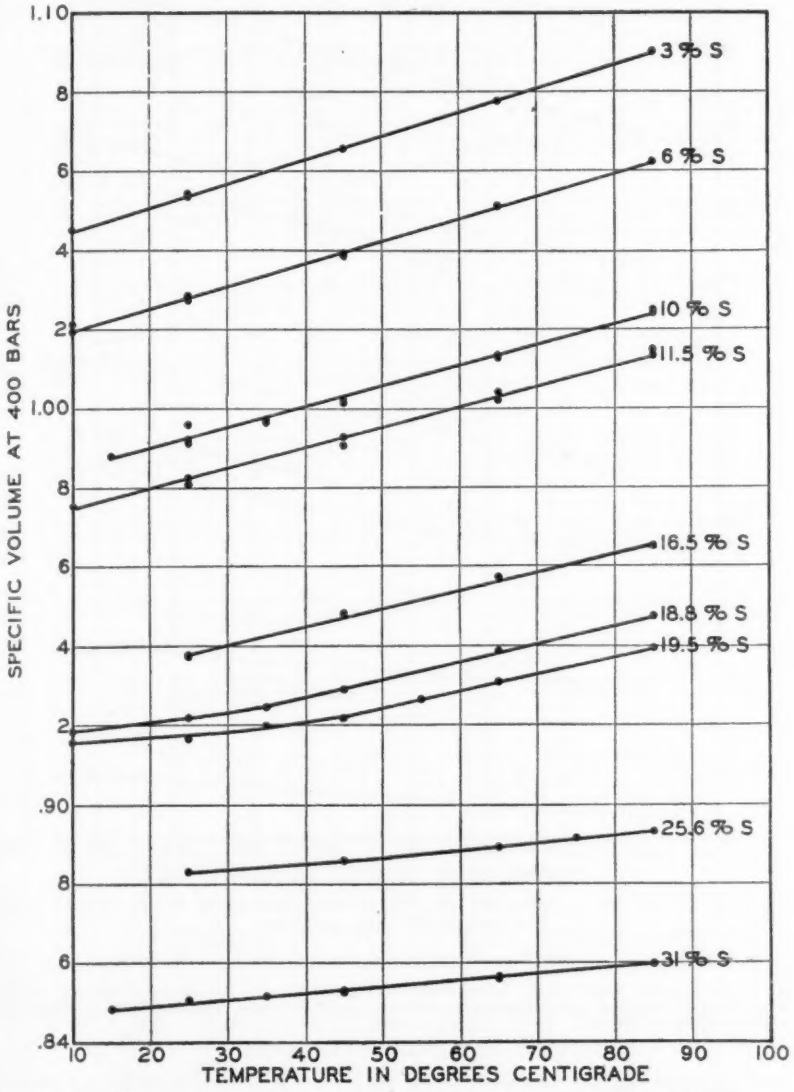

FIGURE 6.—*Effect of temperature on the specific volume of rubber-sulphur compounds at 400 bars pressure.*

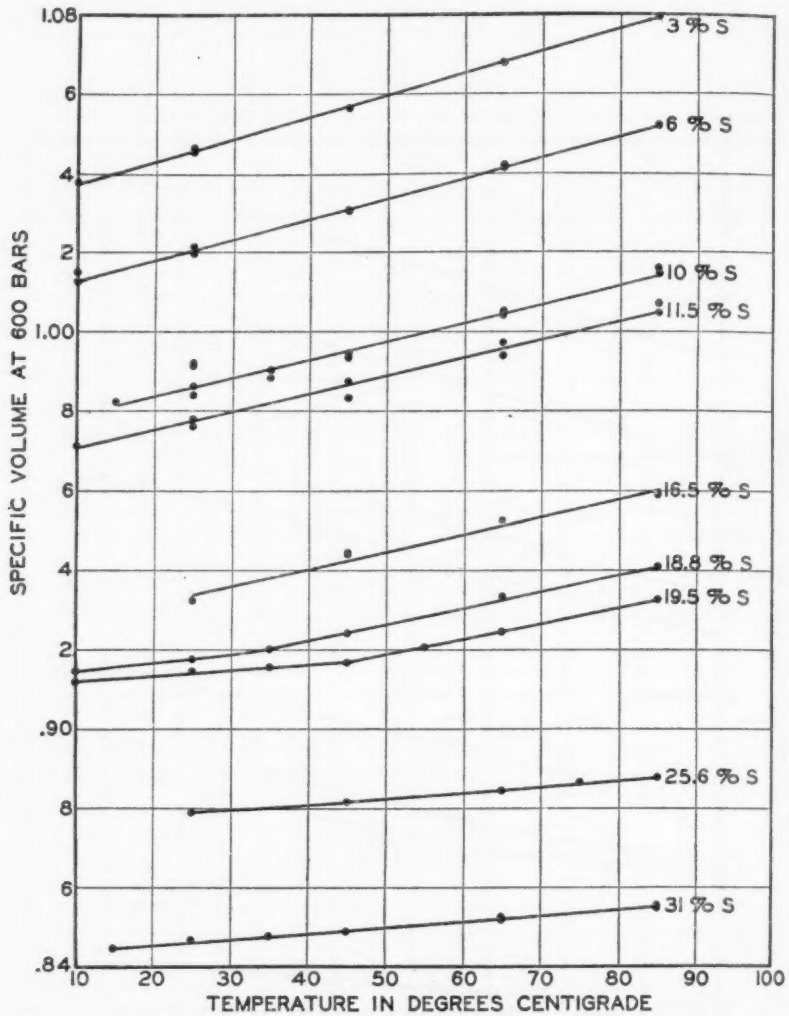

FIGURE 7.—*Effect of temperature on the specific volume of rubber-sulphur compounds at 600 bars pressure.*

TABLE 1.—*Number of sets of pressure measurements made on the various specimens*

[A set of measurements consists of the measurements on a specimen made during one pressure cycle when the pressure is increased from atmospheric pressure to 800 bars and then decreased to atmospheric pressure in steps of 200 bars]

Sulphur content	Number of specimens	Number of sets respectively at								
		10° C	15° C	25° C	35° C	45° C	55° C	65° C	75° C	85° C
Percent										
3	1	1		3		1		1		1
6	1	2		4		2		2		1
10	2		1	4	2	2		3		2
11.5	2	1		2		2		2		2
16.5	1			3		2		1		1
18.8	1	1		1	1	1		1		1
19.5	1	1		2	1	1	1	1		1
25.6	1			1		1		1	1	1
31	2		1	5	2	3		4		3

Thus it will be seen that for 31 percent of sulphur, 5 sets of measurements were made at 25° C on 2 specimens. Yet the values were so close that the differences could not be shown on the figure. On the other hand, 4 sets of measurements were made at 25° C on 2 specimens containing 10 percent of sulphur and the values of specific volume at 800 bars varied from 0.977 to 0.987. These were extreme cases.

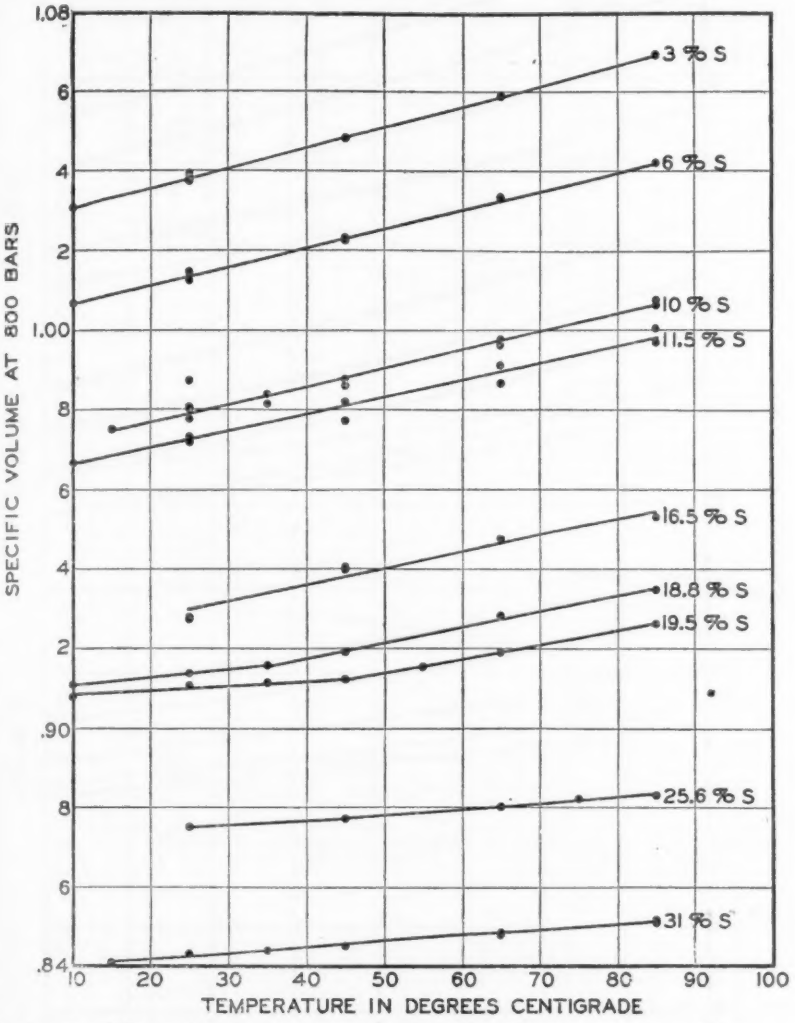

FIGURE 8.—*Effect of temperature on the specific volume of rubber-sulphur compounds at 800 bars pressure.*

The variability in the results was due to lack of stability and reproducibility of the specimens, since the pressure and length measurements could be made with far greater accuracy than the results indicate. The pressure gage was capable of measuring pressure differences of ½ bar and, during calibration, measurements could be reproduced with an error of less than 2 bars. An error of 2 bars would cause a

maximum error in length of about 0.003 cm for the specimen of highest compressibility, which is equivalent to an error in specific volume of

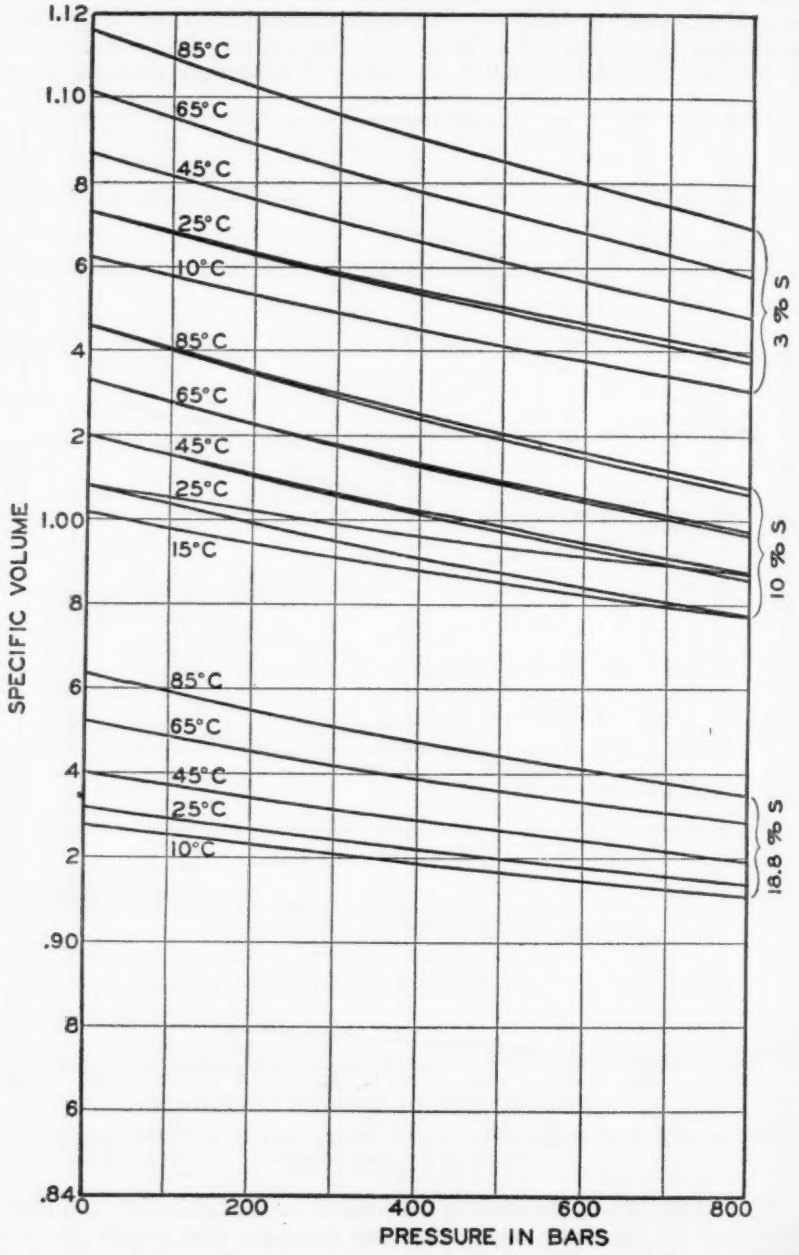

FIGURE 9.—*Effect of pressure on the specific volume of rubber-sulphur compounds containing 3, 10, and 18.8 percent of sulphur.*

about 1 part in 10,000. The measurement of change in length was sufficiently accurate so that a change of 0.0003 cm could be measured.

This is equivalent to a change in specific volume of about 1 part in 100,000. This probably represents the reproducibility of the measurement of change in length since the microscope and specimens were

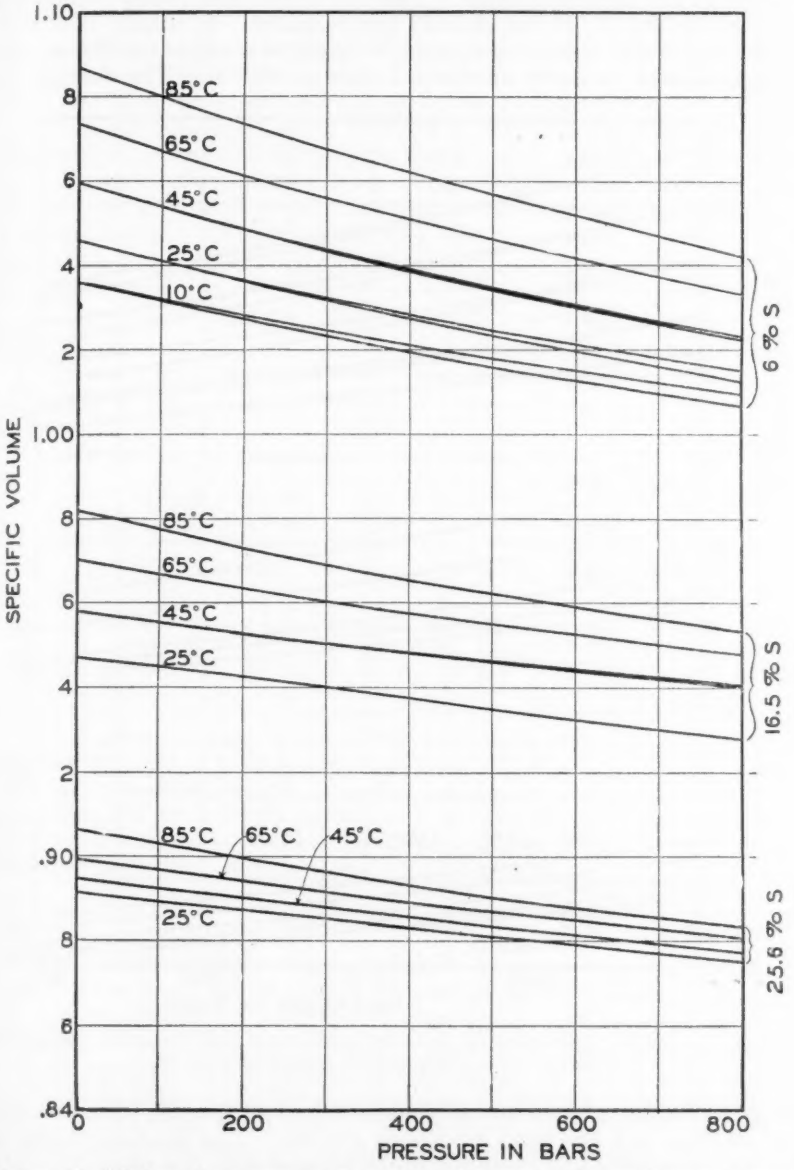

FIGURE 10.—*Effect of pressure on the specific volume of rubber-sulphur compounds containing 6, 16.5, and 25.6 percent of sulphur.*

supported by separate brackets on a brick wall and the changes in pressure in the chamber could not affect the distance between the brackets. As mentioned above, the total length of the specimen was

measured to 0.05 cm. Since the total length was used only in the determination of the change in specific volume with pressure, an error of 0.05 cm in total length introduced an error of less than 3 parts in 100,000 in the specific volume. The temperature of the specimen was kept within 0.1° C of the desired temperature. A change of 0.1° C would produce a maximum change in length of a rod of 0.002 cm or an error in specific volume of about 1 part in 10,000. The sum of the

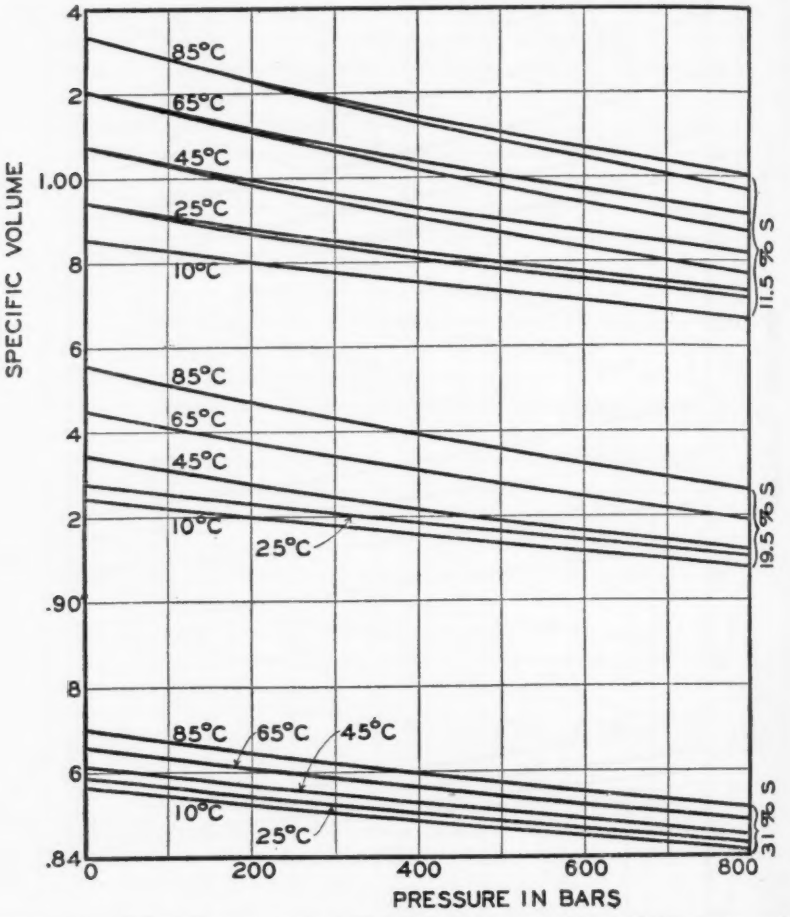

FIGURE 11.—*Effect of pressure on the specific volume of rubber-sulphur compounds containing 11.5, 19.5, and 31 percent of sulphur.*

errors introduced by errors in measurement of length, pressure, and temperature was less than 3 parts in 10,000. The reproducibility of the measurements was therefore much greater than the reproducibility of the results.

The results on the specimens of high sulphur content were much more reproducible than those on specimens of a low sulphur content. There was some variability in the results on the specimen containing 3 percent of sulphur, but not as much as in the results on the specimens containing from 6 to 16.5 percent of sulphur. Instability in

the properties of specimens in this range of sulphur content was noticed in previous work.

3. VOLUME THERMAL EXPANSIVITY

The volume thermal expansivity as obtained from the slopes of the straight lines drawn through the points in figures 4 to 8 are given in table 2. The expansivities are given in two ranges of temperature for the specimens containing 18.8, 19.5, and 25.6 percent of sulphur. The values given for these latter specimens should be considered as only approximate since they were computed for short temperature ranges and usually only one set of measurements was made at a temperature. The values for the other specimens were computed for much larger temperature ranges and more measurements were made.

TABLE 2.—*Volume thermal expansivity*

Sulphur content	Temperature range	Pressure in bars				
		1	200	400	600	800
Percent	°C					
3	10 to 85	6.7×10^{-4}	6.2×10^{-4}	5.8×10^{-4}	5.4×10^{-4}	5.0×10^{-4}
6	10 to 85	6.6	6.0	5.6	5.2	4.8
10	15 to 85	6.4	5.8	5.4	5.1	4.6
11.5	10 to 85	6.6	5.8	5.3	4.7	4.4
16.5	25 to 85	6.2	5.4	5.0	4.5	4.4
18.8	10 to 25	3.0	2.6	2.4	2.1	2.1
18.8	45 to 85	6.2	5.5	5.0	4.6	4.2
19.5	10 to 25	2.5	2.2	1.8	1.6	1.4
19.5	45 to 85	5.7	5.2	4.8	4.3	3.9
25.6	25 to 45	1.9	1.6	1.5	1.4	1.3
25.6	65 to 85	4.0	3.0	2.4	1.9	1.6
31	15 to 65	2.1	2.0	1.9	1.8	1.7

The volume thermal expansivity decreased with both pressure and sulphur content. It was decreased from 6.7×10^{-4} to 5.0×10^{-4} by 800 bars pressure for the specimen containing 3 percent of sulphur, and from 2.2×10^{-4} to 1.7×10^{-4} for the specimen containing 31 percent of sulphur. Values for the latter specimen are about one-third those for the specimens containing small amounts of sulphur.

4. COMPRESSIBILITY

The compressibility was computed from eq 6. The values of α and β for each set of data were computed by means of eq 3 and 4. These coefficients and the specific volume at atmospheric pressure, V_a, were found to be functions of both the temperature and sulphur content of the specimen. From figure 4 it is seen that V_a increases linearly with the temperature for all specimens containing up to 16.5 percent of sulphur. When V_a at 25° C is plotted against the percentage of combined sulphur it is found that it decreases linearly with the percentage of sulphur up to 16.5 percent and from 18.8 to 31 percent. The coefficients α and β increase with the temperature and decrease with the sulphur content. The relationship appears to be linear in each case although there is a wide spread of values in some cases. Assuming linear relationships, it was possible to set up two sets of empirical equations for the values of V_a, α, and β, one for each side of the boundary marking the change in slope of the

curves. For temperatures between 10 and 85° C and for specimens containing up to X percent of sulphur, where X is given by the equation $X=(T+30)/3.7$ and T is the temperature in degrees centigrade, the following equations hold:

$$V_a=1.1015+0.00073(T-25)-X[0.00932+0.000007(T-25)] \quad (7)$$
$$\alpha=-10^{-6}\{53.7+0.258(T-25)-X[1.35-0.00217(T-25)]\} \quad (8)$$
$$\beta=10^{-9}[12.0+0.083(T-25)-0.34X] \quad (9)$$

For temperatures between 10 and 85° C and for specimens containing between X and 32 percent of sulphur, where X is given by the equation $X=(T+42)/3.7$, the following equations hold:

$$V_a=1.0450+0.00038(T-25)-X[0.00601+0.0000064(T-25)] \quad (10)$$
$$\alpha=-10^{-6}\{35.6+0.326(T-25)-X[0.352+0.00672(T-25)]\} \quad (11)$$
$$\beta=10^{-9}[9.1+0.0683(T-25)-0.221X] \quad (12)$$

When these values of V_a, α, and β were substituted in eq 2, equations were obtained which gave the specific volume with an accuracy comparable to the accuracy of the data within the limits specified, with a few exceptions. The computed values of the specific volume agree to about 1 part in 1,000 with experimental values for the specimens containing 3, 6, and 31 percent of sulphur and with part of the experimental values for the specimens containing 10, 11.5, 16.5, 18.8, 19.5, and 25.6 percent of sulphur. In one case only the computed value differs from a measured value by more than 4 parts in 1,000. The computed value for the specimen containing 25.6 percent of sulphur at 85° C is lower than the measured value by about 12 parts in 1,000.

The values for the compressibility may be readily determined within the limits specified from eq 6 by substituting in it the correct values of α and β as computed from eq 8 and 9 or 11 and 12. The values for the specimens containing 3 and 31 percent of sulphur are shown in figure 12. These were the extreme values of the compressibility which were obtained.

VI. DISCUSSION

The method used for the determinations of the specific volumes and compressibilities of rubber was such that the precision of measurement was much higher than the reproducibility of results. As mentioned above the measurements could be made to less than 3 parts in 10,000. The reproducibility of the results was such that during any set of measurements through a pressure cycle at a fixed temperature the values obtained for increasing pressures agreed with those for decreasing pressures to about 2 parts in 10,000. However, if a duplicate specimen were made or if the specimen were taken through a cycle of temperatures and measured again at the original temperature, the measurements in many cases did not agree closer than about 2 or 3 parts in 1,000 at 800 bars. In one case the measurements on different specimens did not agree closer than 1 part in 100. The values of specific volume are therefore probably accurate to 0.2 or 0.3 percent in most cases and in any case to 1 percent.

There is very little published information on the compressibility of rubber with which to compare the present work. Adams and Gibson

have measured 3 samples of rubber at pressures above 1,000 bars.[13] Only one of these had a composition falling in the group used in the present investigation. The extension of the curve of Adams and Gibson for a specimen containing 10 percent of sulphur gives a compressibility of about 29.4×10^{-6} at 800 bars. The compressibility for the same composition obtained from eq 6, using the values of α and β computed from eq 8 and 9, is 26.4×10^{-6} at 25° C. The values

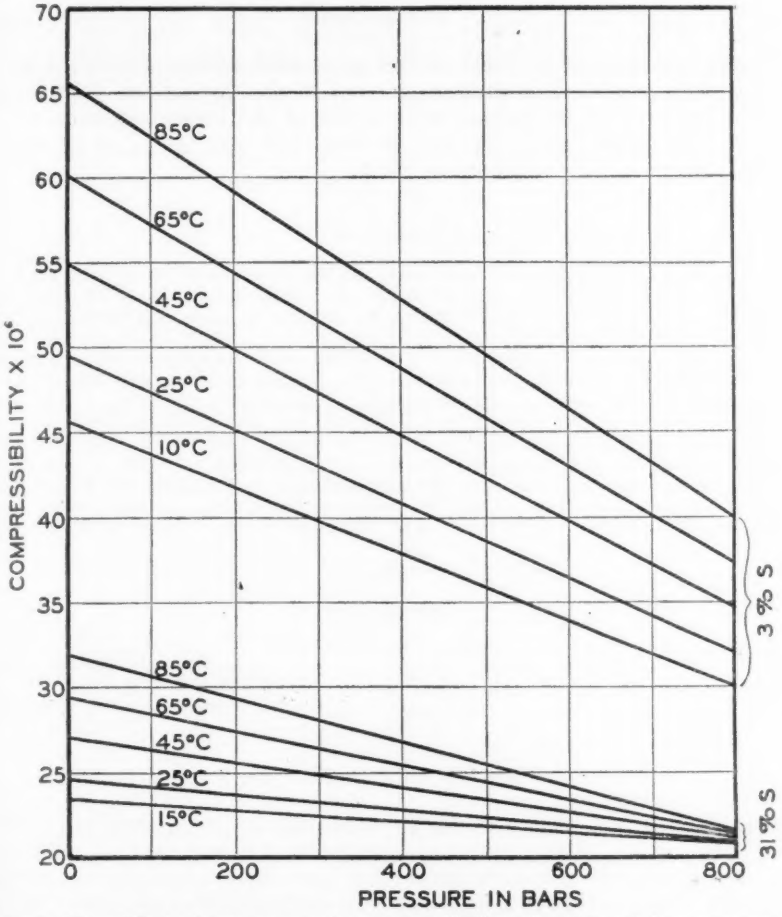

FIGURE 12.—*Effect of pressure on the compressibility of rubber-sulphur compounds containing 3 and 31 percent of sulphur.*

obtained for the individual sets of data, where α and β were computed from eq 3 and 4, varied from 15.5 to 29.4×10^{-6}. The value given by Adams and Gibson is then about the same as the highest value obtained by the author. But the change of the compressibility with pressure obtained by the author is very much greater than that given by the curve of Adams and Gibson.

The specific volumes of rubber-sulphur compounds at atmospheric pressure have been determined by Kimura and Namikawa.[14] They

[13] J. Wash. Acad. Sci. **20**, 213 (1930).
[14] J. Soc. Chem. Ind., Japan, **33** (supplemental binding) 196B (1929).

used specimens made from crude rubber. The values which they reported are in agreement with those given here for specimens containing up to 16.5 percent of sulphur. For specimens containing larger amounts of sulphur the values of specific volume given by them are lower than those given here. The "knees" which they describe were observed by the author, but they were generally found to come at lower temperatures than those reported by Kimura and Namikawa.

Acknowledgment is made of the important assistance which A. D. Cummings rendered in preparing most of the specimens used in this investigation and in helping with some of the measurements.

WASHINGTON, November 10, 1934.

U. S. Department of Commerce National Bureau of Standards

RESEARCH PAPER RP761

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INFLUENCE OF MAGNESIUM SULPHATE ON THE DETERIORATION OF VEGETABLE-TANNED LEATHER BY SULPHURIC ACID

By Roy C. Bowker, Everett L. Wallace, and Joseph R. Kanagy

ABSTRACT

The influence of magnesium sulphate on the deterioration of chestnut and quebracho leathers was studied. Samples of both leathers, which contained varying percentages of sulphuric acid, and comparable leathers, which contained similar amounts of sulphuric acid and approximately 5 percent of magnesium sulphate, were examined. The addition of magnesium sulphate caused an increase in the pH of the leathers. Deterioration was determined by measuring the change in strength after 6, 12, 18, and 24 months. Deterioration of the samples aged for 24 months was also determined by measuring the extractable nitrogen. The results by both physical and chemical methods show that, for the same percentage of acid, the leathers containing magnesium sulphate deteriorated less. However, it was found that deterioration was a function of pH rather than of actual acid content. All leathers below pH 2.8 showed serious deterioration, while the percentage of acid required to cause it varied from .75 to 1.75 percent.

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I. INTRODUCTION

The production of commercial leather requires the use of many materials in addition to skins and tanning agents. One of these materials, sulphuric acid, has long been used for bleaching in connection with the manufacture of vegetable-tanned leather. Enough acid may be introduced during the process to cause serious damage to the leather within a short time. For this reason more attention has been given to limitations on the permissible mineral acidity in leather than to any other criterion in common use for judging quality. The limiting values established in specifications have been dictated largely by experience gained from observations of the behavior of leather in actual service, instead of by accurate information as to the actual quantitative amounts which would bring about deterioration. The latter has been the object of research conducted at the Bureau.[1]

[1] R. C. Bowker and E. L. Wallace. *The influence of* pH *on the deterioration of vegetable-tanned leather by sulphuric acid.* BS J. Research **10,** 559 (1933) RP548.

Thirteen lots of leather, specially prepared and treated with varying percentages of sulphuric acid, were studied. The results showed that the percentage of sulphuric acid which would cause serious deterioration in two years varied from 0.3 to 1.8 for the different leathers. Effective acidity, as indicated by pH measurements, was determined for all of these leathers. It was found that deterioration bore a more definite relation to the pH of the leather than to the percentage of acid in it. In all cases deterioration started near or at pH 3.

Among the materials considered likely to have an influence on the determination of the acidity of leather and on its deterioration by sulphuric acid are magnesium sulphate, glucose, sulphonated oils, sulphite cellulose extracts, and synthetic tanning materials. This report deals with the influence of magnesium sulphate which is largely used in the finishing of sole and black harness leathers.

II. MATERIALS AND METHODS USED

One lot of leather (no.19) was tanned with ordinary quebracho extract and another lot (no. 20) was tanned with powdered chestnut extract. The chemical analyses of these leathers are given in table 1.

TABLE 1.—*Chemical analyses of leathers*

[Results expressed in percentage excepting for degree of tannage and pH values]

Tanned with	Lot no. 19, quebracho	Lot no. 20, chestnut
Water solubles	8.10	8.53
Hide substance	48.87	50.40
Grease (P. E. E.)	3.50	3.25
Moisture	10.97	11.30
Insoluble ash	.10	.24
Combined tannin [1]	28.46	26.28
Total	100.00	100.00
Degree of tannage [2]	58.20	52.10
Soluble tans	6.59	7.11
Soluble nontans	1.51	1.42
Total ash	.11	.25
Acid (P and S)	.04	[3].07
pH value [4]	5.21	3.90

[1] Determined by difference.
[2] Ratio of combined tannin to hide substance.
[3] Alkalinity.
[4] 4.9 g of dry leather in 100 ml of distilled water.

Seven hides were used in preparing the leather for each lot. Twenty-four 7 by 16 inch samples were cut from each hide, as shown in figure 1, making a total of 168 for each lot. Each sample was given a code letter designating its location on the hide and a code number designating the hide from which it was cut. Each sample was of sufficient size to permit 12 test specimens to be cut from it as required. This division of a sample is indicated at the lower left in figure 1. The first step was to determine the original tensile strength of the leather. Two specimens, of the shape and size shown in figure 2, were cut from each sample for this purpose.

The 168 samples from each lot were then divided into 8 groups of 21 each. They were treated in a manner to secure 4 groups con-

taining 0, and approximately ¾, 1¾, and 2½ percent of sulphuric acid, respectively. The four remaining groups were similarly prepared, but had also approximately 5 percent of magnesium sulphate ($MgSO_4 \cdot 7H_2O$) added. The tensile strength was determined immediately after the treatments by testing two more specimens from each block.

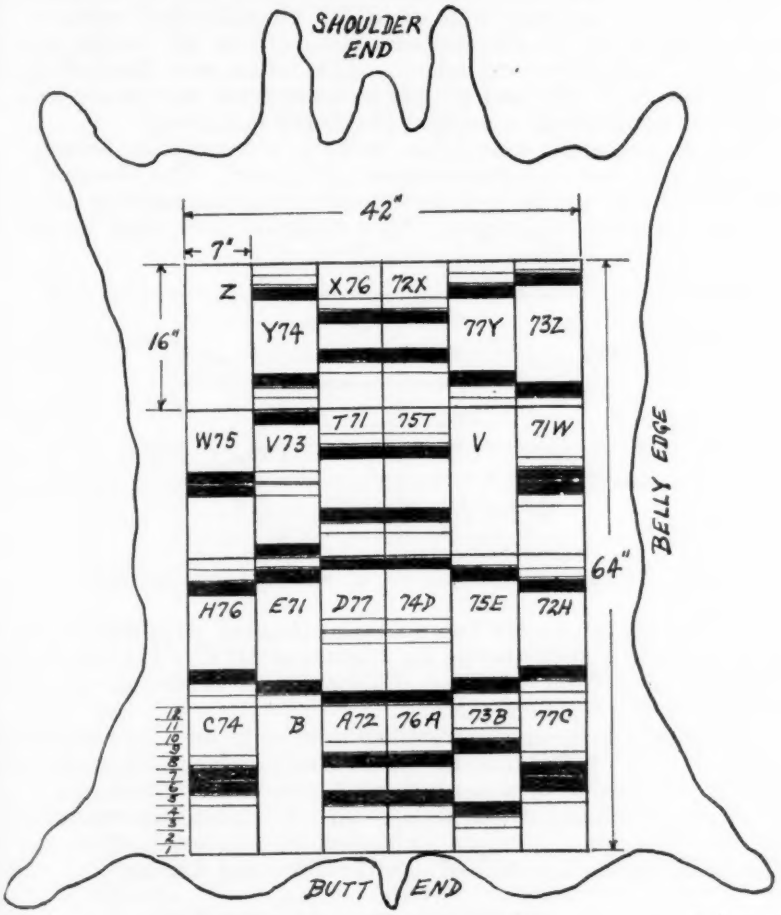

FIGURE 1.—*Method of sampling.*

The leather was then stored at 70° F, and 65 percent relative humidity and its strength determined after 6, 12, 18, and 24 months.

Each group of samples was selected in a manner to make it, as nearly as possible, representative of the lot as a whole. A typical selection of a group of samples is shown in figure 1. It will be noted that samples from each of the hide locations and from each hide were included. The distribution was governed by a statistical study of the strengths of approximately 1,500 samples previously examined in other phases of the subject.

The strength of any group at any period was determined as the average result of 42 tests since two specimens were tested from each sample at each period. Shaded spaces in figure 1 show the

distribution of the specimens used for determining the original strength of the chestnut leather group which contained 2.4 percent of acid. The open spaces, which in most cases are adjacent to the shaded ones, show the distribution of the specimens used for determining the strength of the group after the 24-month aging period. The change in strength during aging was taken as a measure of the deterioration.

The ALCA methods were used for all analytical work on the leather excepting for the determination of the pH values and the amount of sulphuric acid added. The latter was checked by the oxygen bomb method and the pH measurements were made with the hydrogen electrode by a method previously described.[2]

This investigation shows that there is a correlation between the physical and chemical deterioration of leather. The chemical deterioration of the samples aged for 24 months was measured by determining the extractable nitrogen. Two methods were used as follows:

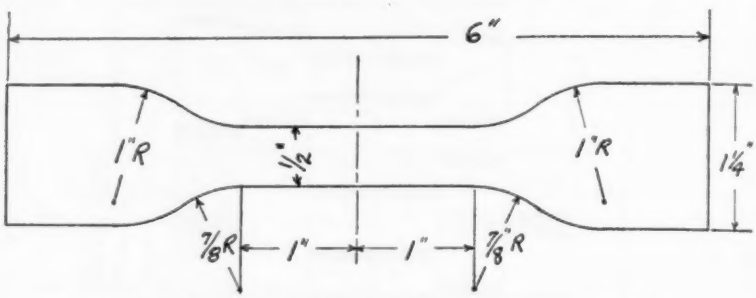

FIGURE 2.—*Shape and size of specimen for strength tests.*

(A) Twenty g of each sample was extracted with 200 ml of distilled water in a shake-bottle for 3 hours at 25° C. The solution was filtered and a total nitrogen determination made on an aliquot portion by the Kjeldahl method.

(B) Fifteen successive extractions were made on a 3 g sample using water at 50° C. The time for each extraction was 10 minutes and a fresh 20 ml portion of water was used each time. The same sample was then given similar extractions with 0.1 *N* sodium carbonate at 25° C. The total nitrogen extracted was determined as in (A). This is similar to the method used by Frey and Clarke.[3]

III. RESULTS AND DISCUSSION

The percentage of sulphuric acid added, the percentage of magnesium sulphate added, the original pH values and the pH values of the samples after aging 24 months are given in table 2.

[2] E. L. Wallace and John Beek, Jr. *A comparison of the quinhydrone and hydrogen electrodes in solutions containing tannin.* BS J. Research **4**, 737 (1930) RP176.

[3] *The decay of bookbinding leathers.* J. Am. Leather Chem. Assn. **26**, 461 (1931).

TABLE 2.—pH *values and percentages of acid and magnesium sulphate for the different groups of samples.*

[Lot no. 19, Quebracho]

Sulphuric acid	Magnesium sulphate	pH	
		Original	24 months
Percent	*Percent*		
0.0	----------	4.90	4.96
.7	----------	3.26	3.32
1.6	----------	2.67	2.88
2.2	----------	2.28	2.60
Quebracho With Magnesium Sulphate			
0.0	4.7	4.97	4.91
.8	4.6	3.52	3.60
1.5	4.9	2.81	2.98
2.3	4.9	2.43	2.63
Lot no. 20, Chestnut			
0.0	----------	3.79	3.80
.8	----------	2.87	2.98
1.7	----------	2.47	2.70
2.4	----------	2.18	2.44
Chestnut With Magnesium Sulphate			
0.0	5.0	3.87	3.94
.8	5.2	3.16	3.26
1.7	4.9	2.64	2.80
2.5	5.3	2.33	2.52

The results in table 2 were from a composite of the 21 samples in each group. A consideration of the original pH values shows that the immediate effect of adding magnesium sulphate is to increase the pH of the leather. It is of interest to note that the pH of all groups containing acid increased during aging.

It was previously stated that the strength of any group of samples at any period was the average of the strengths of 42 specimens. Figure 3 shows the strengths of the individual test specimens illustrated in figure 1. The upper values are for the shaded spaces and were used in determining the original strength of the group. The lower values are for the open spaces and were used in determining the strength of the group after treatment with acid and aging for 24 months. The small numbers near the points designate the locations of the specimens on the samples. In this case, the original strength and the strength after 24 months were 3,995 and 2,020 lb/in.^{2} The loss of strength on aging was approximately 50 percent. Each point in figures 4, 5, and 6 was obtained in this manner. It is of interest to note that the samples from different hides, but from similar hide locations, usually did not vary in strength more than 10 percent.

The results for the deterioration of the leathers are shown in figure 4 for all aging periods as a function of the percentage of sulphuric acid. In graph A the results are shown just as measured. It will

be noted that there is some change in strength for the control samples which contain no acid. In order to indicate the deterioration caused by the acid, all results have been corrected for the change in strength of the controls and these results are shown in graph B.

The small figures near the points denote the aging period. Those marked with a zero are the results of tests made immediately after

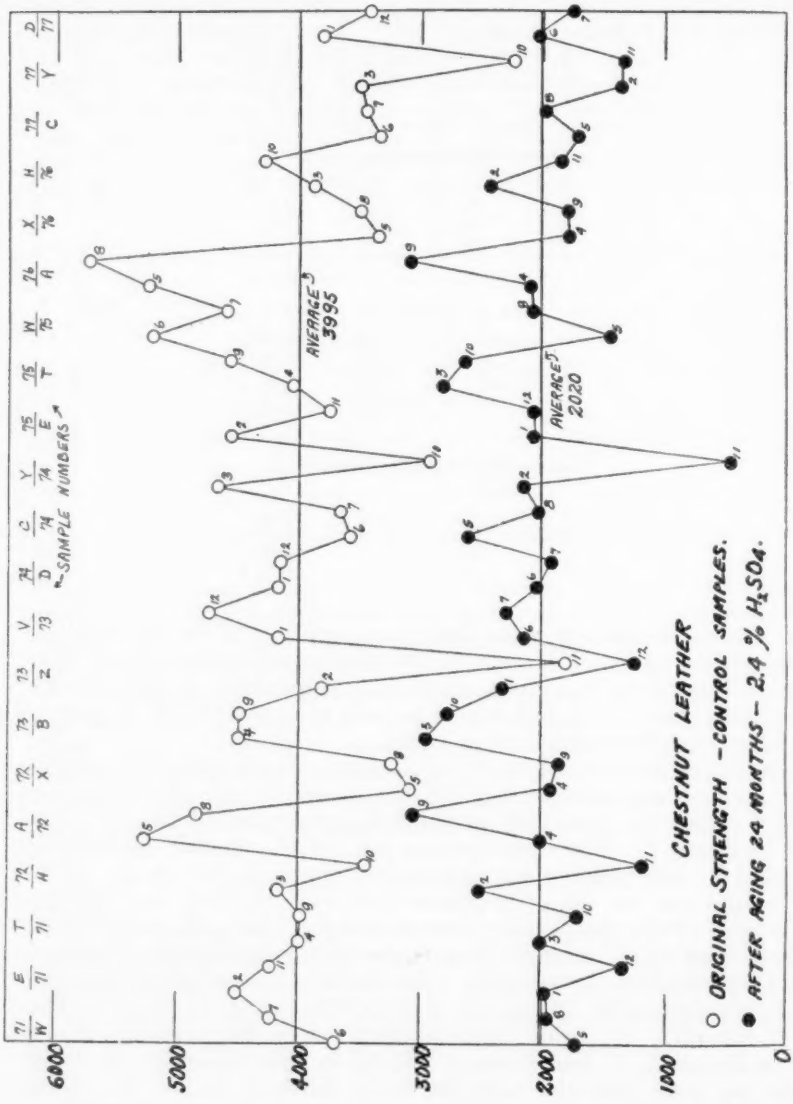

FIGURE 3.—*Tensile strengths in pounds per square inch of individual specimens from a group of chestnut leather samples to determine the original strength of the group.*

Also strengths of individual specimens from the same samples after treatment with 2.4 percent of sulphuric acid and aging for 24 months.

treatment. In most cases the immediate effect of the treatment is to cause a loss in strength which increases with the amount of acid. One exception is the quebracho leather containing magnesium sulphate. This indicated deterioration is considered to be real in view of the fact that the control samples were wetted with water to compensate for any effect due to wetting with acid solutions. It is con-

sidered in the final results as a part of the total deterioration. It is evident that, for any given acid content, the leather containing

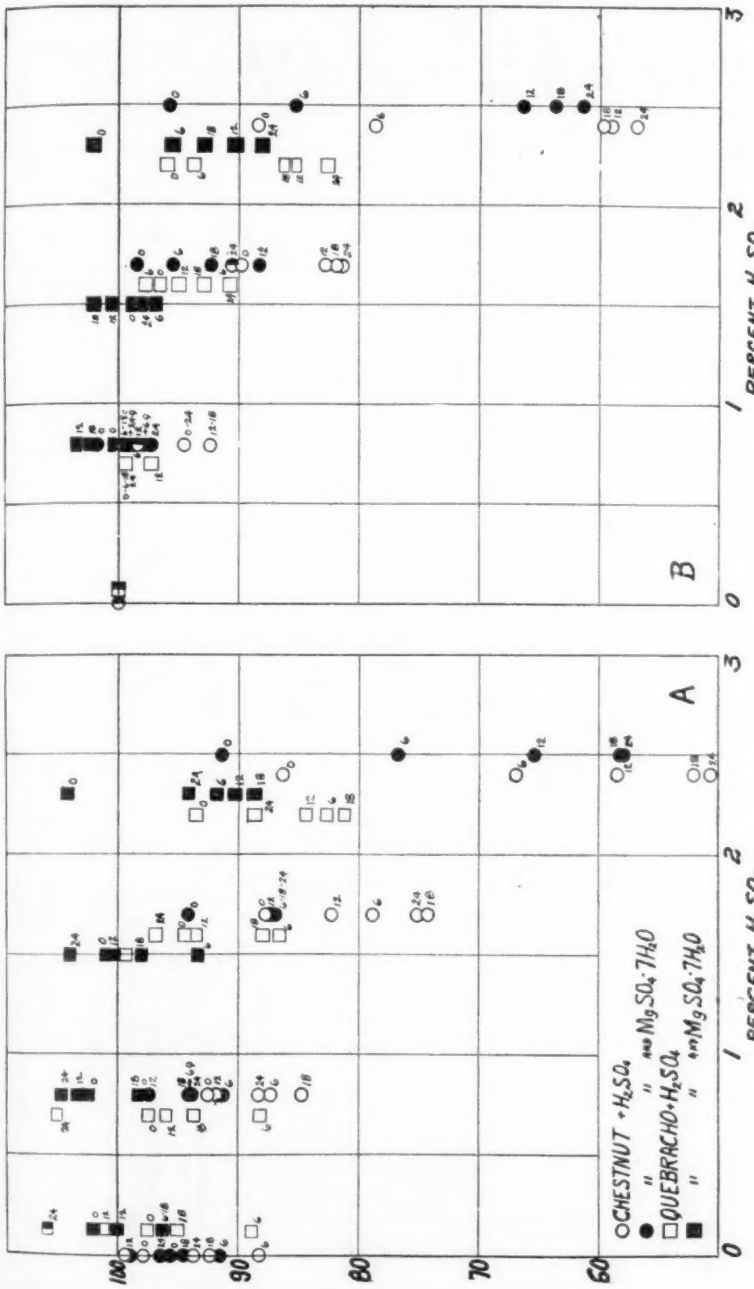

FIGURE 4.—*Deterioration of leathers for all aging periods as a function of sulphuric acid.*

Ordinates show percentage of original strength.
Graph A.—Results as measured.
Graph B.—Results corrected for change in strength of the control (0-acid) samples.
Small figures near points denote the aging periods in months.

magnesium sulphate did not deteriorate to as great a degree as the leather which did not contain this salt. The results also show little difference in deterioration after 12 and 24 months. The quebracho

leather, for any given acid content, did not deteriorate as much as the chestnut leather.

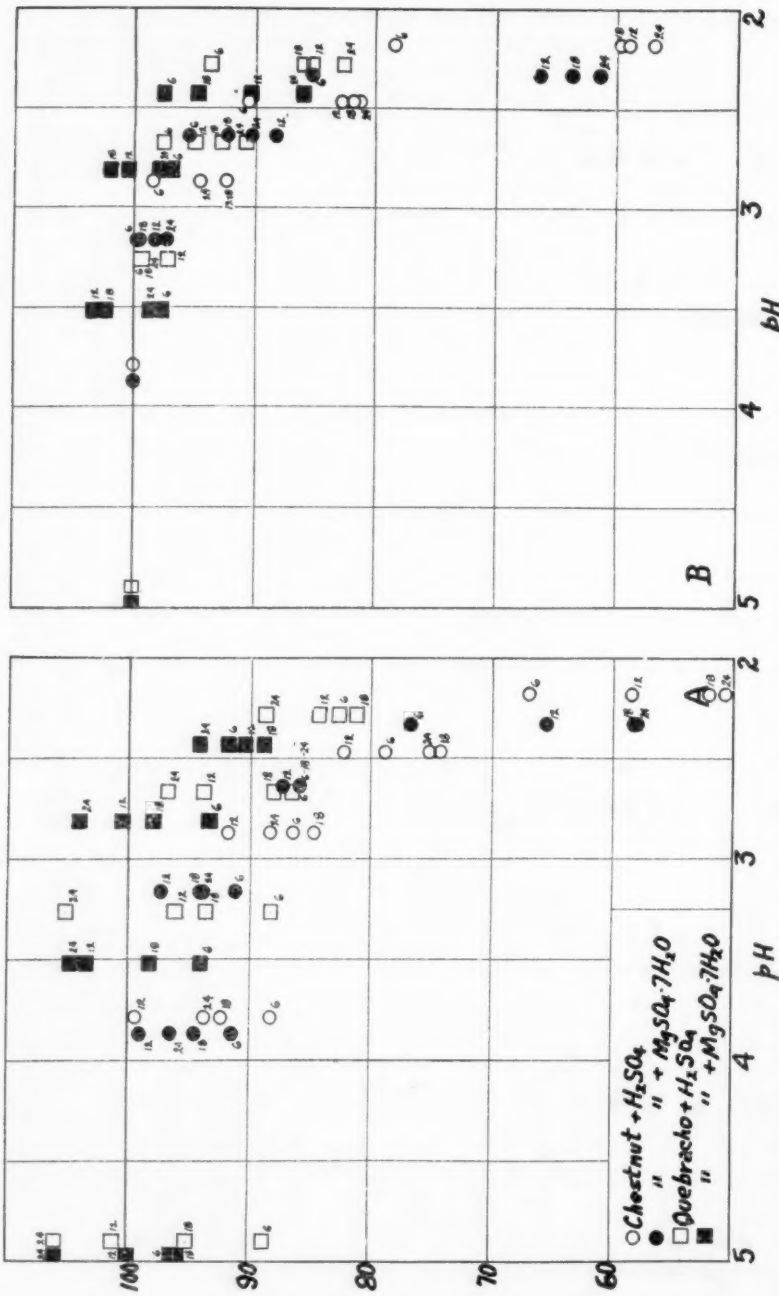

FIGURE 5.—*Deterioration of leathers for all aging periods as a function of original* pH.

Ordinates show percentage of original strength.
Graph A.—Results as measured.
Graph B.—Results corrected for the change in strength of the control (0-acid) samples.
Small figures near the points denote the aging periods in months.

The results for the deterioration of the leathers as a function of the original pH are given in figure 5. The results as measured

are shown in graph A and, as corrected, in graph B. These values show a marked difference when compared with those expressed as a

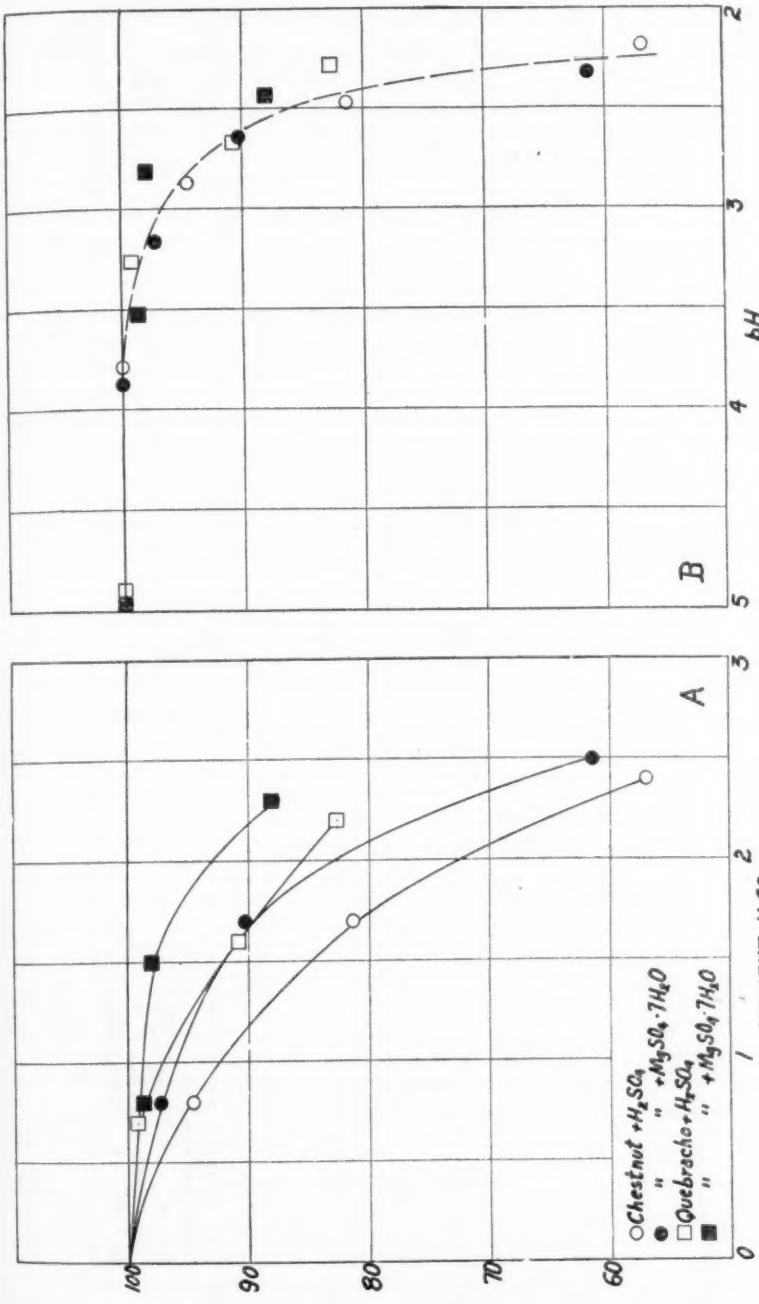

FIGURE 6.—*Deterioration of the leathers aged for 24 months.*

Ordinates show percentage of original strength. Graph A.—As a function of the percentage of acid. Graph B.—As a function of the original pH.

function of the percentage of acid. In the latter case, figure 4, the samples containing magnesium sulphate very definitely showed less

deterioration than those containing none of this salt. There is no such definite trend shown when the results are expressed as a function

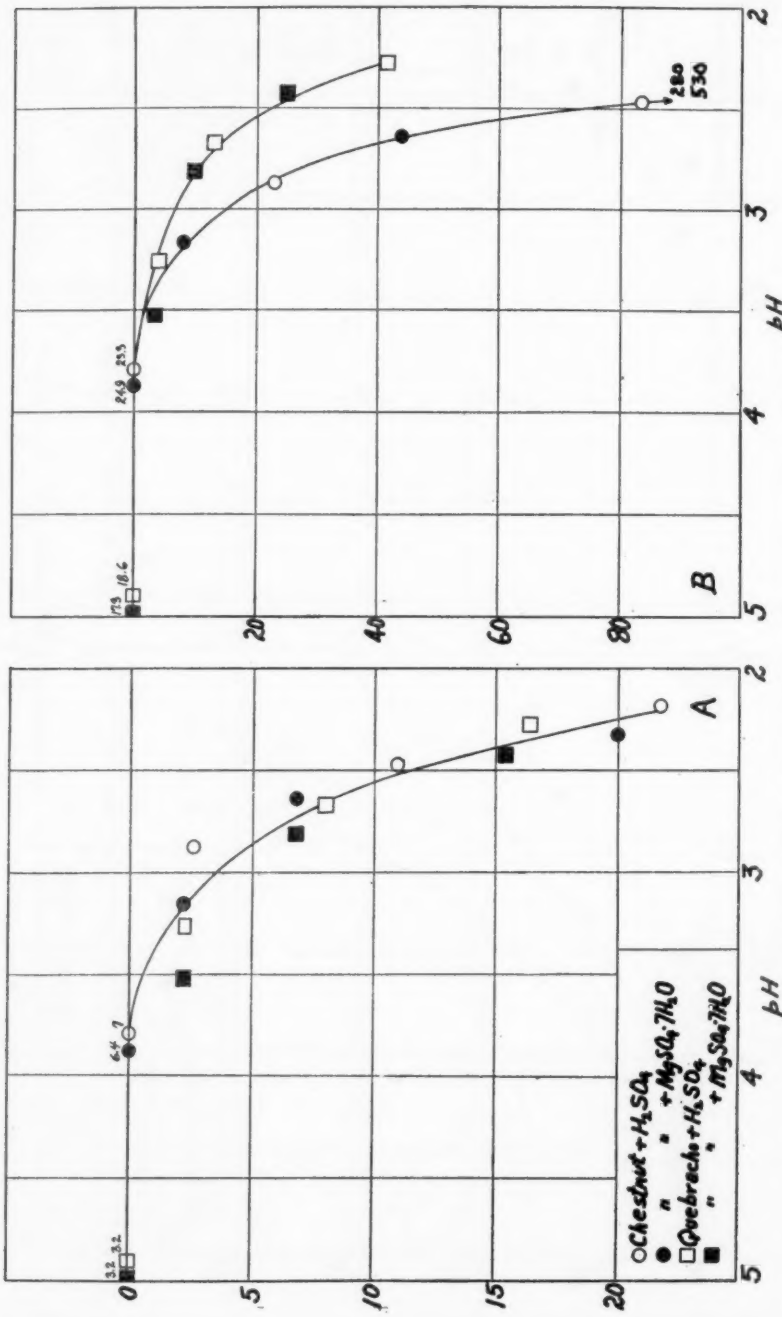

FIGURE 7.—*Chemical deterioration of the leathers aged for 24 months.*

Small figures on points on base line denote correction made for nitrogen extracted from the control (0-acid) samples. Ordinates show milligrams of soluble nitrogen. Graph A.—Nitrogen extracted with water at 25° C. Graph B.—Nitrogen extracted with water at 50° C and then with 0.1 *N* Na_2CO_3 at 25° C.

of the original pH values of the leathers. This is clearly illustrated by the results, figure 6, for the samples aged for 24 months. Graph

A shows the deterioration of the leathers as a function of the percentage of acid. The leathers containing magnesium sulphate have clearly lost less strength than the companion leathers containing no salt. The same results are given in graph B as a function of the original pH values of the leathers. Deterioration of the leathers appears to start near pH 3 and to be serious for all groups having pH values below 2.8. The data presented in figure 6 definitely demonstrate that the pH value of leather is a more reliable measure of harmful acidity than a determination of the percentage of acid. It may be safely considered that a leather has deteriorated only slightly if it possesses 95 percent of its original strength after aging for 2 years. Graph A, figure 6, shows that the percentage of sulphuric acid required to meet this condition varies from .75 to 1.75 for these leathers. By contrast, all groups having a pH value of below 2.8, graph B, show more than 5 percent loss in strength.

The results for the determinations of the soluble nitrogen extracted from the leathers aged for 24 months are presented in figure 7. All results are expressed in milligrams of nitrogen extracted from a twenty-gram sample and as a function of the pH. The similarity between these chemical results and the physical results, graph B, figure 6, will be noted. The amount of nitrogen extractable increases sharply for all leathers below pH 3 as does the loss in strength on aging.

The hydrolysis of chestnut and quebracho leathers in sulphuric-acid solutions as influenced by magnesium sulphate was previously studied.[4] It was shown that the hydrolysis was retarded, when magnesium sulphate was present, over the range of acid concentrations used. It is of interest to observe that the influence of magnesium sulphate on the hydrolysis of leather in acid solutions and on the deterioration of leather in the dry state is strikingly similar for any given acid condition.

IV. SUMMARY

The results of this work may be summarized as follows:

1. The addition of magnesium sulphate to leather caused an increase in pH, that is, a decrease in the concentration of hydrogen ions.

2. Leathers containing magnesium sulphate deteriorated less in 2 years than comparable leathers having the same percentage of sulphuric acid and no magnesium sulphate.

3. Deterioration of the leathers, as measured by both physical and chemical methods, was found to be a function of pH rather than of the percentage of acid.

4. The leathers started to deteriorate near pH 3 and deterioration was of major proportions for all leathers having a pH below 2.8. The percentage of acid required to cause similar deterioration varied from 0.75 to 1.75 percent.

WASHINGTON, December 19, 1934.

[4] E. L. Wallace and J. R. Kanagy. *The influence of sodium chloride and magnesium sulphate on the hydrolysis of leather by sulphuric acid.* J. Am. Leather Chem. Assn. **4**, 186 (1933).

Small figures on points on base line denote correction made for nitrogen extracted from the control (0-acid) samples. Ordinates show milligrams of soluble nitrogen. Graph A.—Nitrogen extracted with water at 25° C. Graph B.—Nitrogen extracted with water at 25° C and then with 0.1 *N* Na_2CO_3 at 25° C.

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INDEX OF REFRACTION, DENSITY, AND THERMAL EXPANSION OF SOME SODA-ALUMINA-SILICA GLASSES AS FUNCTIONS OF THE COMPOSITION

By Conrad A. Faick, John C. Young, Donald Hubbard, and Alfred N. Finn

ABSTRACT

The index of refraction and density of 44 soda-alumina-silica glasses and the thermal expansion of 29 of the glasses were determined. The composition of the glasses, which varied from 1 to 10 percent alumina, 19 to 45 percent soda, and 50 to 78 percent silica, was determined by chemical analyses. The data are given in tabular form and graphs are presented from which the index of refraction, density, critical and softening temperatures, and the linear thermal expansion from room temperature to these temperatures can be predicted with reasonable accuracy for any glass in the series studied. The substitution of alumina for soda and silica in the proper ratios will give glasses having the same refractivity, density, and expansion as the original soda-silica glass; these ratios are approximately 1:1.8, 1:1, and 1:1.5, respectively. On the expansivity for a given temperature range the effect is about the same same as that of silica.

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I. INTRODUCTION

Certain relations between chemical composition of some soda-lime-silica glasses and their index of refraction, density, and thermal expansion are given in three previous reports.[1] This report is the result of a study of similar relations for a series of soda-alumina-silica glasses.

In the first report the purpose of the general study was given, the method of making glasses was described, essential parts of the methods of chemical analysis and the condition of annealing were outlined, and the method of measuring index of refraction was detailed. Density determinations were discussed in the second report and the third report contained the necessary details for measuring thermal expansions.

[1] C. A. Faick and A. N. Finn. *The index of refraction of some soda-lime-silica glasses as a function of the composition.* J. Am. Ceram. Soc. **14,** 518 (1931). F. W. Glaze, J. C. Young, and A. N. Finn. *The density of some soda-lime-silica glasses as a function of the composition.* BS J. Research **9,** 779 (1932) RP507. B. C. Schmid, A. N. Finn, and J. C. Young. *Thermal expansions of some soda-lime-silica glasses as functions of the composition.* BS J. Research **12,** 421 (1934) RP667.

Since the soda-alumina-silica glasses were made and handled in the same way as the soda-lime-silica glasses, descriptions of the methods involved will not be repeated in this report.

II. SCOPE

Forty-four glasses, containing approximately from 1 to 10 percent alumina, 50 to 78 percent silica, and 19 to 45 percent soda, were made and tested. The index of refraction for sodium light, and the density at approximately 20° C were determined for all the glasses; the thermal expansion and critical and softening temperatures were determined for only 29 of them. All the data obtained, including chemical composition, are given in table 1.

TABLE 1.—*Compositions, observed indices of refraction, densities, critical and softening temperatures, and linear thermal expansions between the indicated temperatures*[1]

Glass no.	Composition			Index of refraction[2] n_D observed	Density[2] observed at 20° C	Critical temperature	Linear expansion 25° C to critical temperature	Softening temperature	Linear expansion 25° C to softening temperature
	SiO_2	Na_2O	Al_2O_3						
	%	%	%			°C	μ/cm	°C	μ/cm
101	50.32	46.82	2.86	1.5162(−1)	2.5605(+15)				
102	50.57	42.43	7.00	1.5154(−2)	2.5495(+10)	447(+2)	82.0(−1)	489(+2)	94.5(0)
103	50.86	42.43	6.71	1.5155(+3)	2.5484(+5)	447(+3)	83.7(0)	492(+7)	98.5(+4)
104	50.89	39.54	9.57	1.5150(+2)	2.5410(−3)	455(0)	82.0(0)	501(0)	99.0(+6)
105	50.95	44.17	4.88	1.5157(0)	2.5533(+13)	435(−1)	84.2(0)	472(0)	94.8(0)
106	55.25	34.92	9.83	1.5116(+2)	2.5182(−5)	477(+4)	76.0(−1)	517(0)	86.2(−2)
107	55.56	37.50	6.94	1.5120(−1)	2.5259(+11)	457(0)	77.2(0)	501(+1)	90.5(+2)
108	55.66	34.66	9.68	1.5110(−2)	2.5163(−7)				
109	55.80	41.32	2.88	1.5130(+2)	2.5364(+22)	440(+2)	79.0(0)	484(+9)	91.5(0)
110	55.86	39.23	4.83	1.5124(0)	2.5304(+16)	445(−2)	78.9(+1)	492(+3)	93.0(+4)
111	60.45	29.71	9.84	1.5072(+1)	2.4897(−12)				
112	60.68	34.46	4.86	1.5088(+2)	2.5032(−8)	465(+3)	74.0(0)	504(0)	86.2(+2)
113	60.76	36.42	2.82	1.5092(+1)	2.5111(+19)	455(+2)	75.4(−1)	491(+1)	86.0(0)
114	60.78	38.13	1.09	1.5094(0)	2.5168(+30)				
115	60.88	29.53	9.59	1.5070(+2)	2.4891(−4)	495(+2)	68.8(−1)	530(−7)	79.4(−2)
116	60.97	32.46	6.57	1.5073(−5)	2.4977(+3)				
117	64.78	34.20	1.02	1.5059(−2)	2.4916(−13)	458(+1)	72.2(0)	500(+10)	84.5(+3)
118	65.10	24.94	9.96	1.5032(+1)	2.4615(−5)	510(+2)	65.2(+1)	561(0)	76.8(0)
119	65.70	29.55	4.75	1.5049(+7)	2.4749(−6)	475(−3)	70.4(+2)	530(+12)	84.6(+8)
120	65.88	24.69	9.43	1.5025(−1)	2.4580(+1)	505(−3)	62.0(−1)	550(−10)	72.0(−3)
121	65.90	31.23	2.87	1.5052(+6)	2.4830(+25)	470(+3)	72.0(+3)	500(−5)	85.2(+7)
122	65.92	31.67	2.41	1.5047(0)	2.4814(−3)				
123	66.36	31.61	2.03	1.5050(+5)	2.4805(+3)	470(+6)	67.2(−2)	517(+14)	83.0(+5)
124	67.21	28.34	4.45	1.5026(−4)	2.4653(−13)				
125	67.75	29.30	2.95	1.5030(−3)	2.4683(+1)				
126	68.10	25.15	6.75	1.5018(+3)	2.4539(+11)	493(−3)	63.0(0)	540(−3)	74.2(0)
127	68.78	25.45	5.77	1.5006(−1)	2.4534(+18)				
128	69.99	29.04	0.97	1.5020(+4)	2.4612(+10)	470(0)	69.6(+4)	515(+10)	81.5(+7)
129	70.11	27.86	2.03	1.5015(−1)	2.4572(+17)	476(+2)	65.1(0)	520(+8)	76.5(+4)
130	70.30	24.84	4.86	1.5003(+3)	2.4445(−1)	490(0)	60.3(−1)	535(−2)	70.4(−1)
131	70.63	24.45	4.92	1.5000(+4)	2.4428(+7)	490(−2)	61.0(0)	541(+3)	76.9(+6)
132	70.84	26.31	2.85	1.5007(+6)	2.4509(+34)				
133	70.88	22.63	6.49	1.4988(+1)	2.4343(−11)	500(−1)	57.7(0)	553(0)	69.5(0)
134	71.08	19.60	9.32	1.4970(−2)	2.4214(−10)	522(0)	54.0(+1)	570(−5)	67.5(0)
135	73.15	25.87	0.98	1.4984(+2)	2.4396(+11)				
136	73.81	21.50	4.69	1.4957(−2)	2.4181(−5)	502(+2)	53.0(−2)	552(+4)	64.0(−2)
137	74.75	24.27	0.98	1.4970(+3)	2.4277(+7)				
138	74.88	22.12	3.00	1.4955(−2)	2.4162(−1)				
139	74.95	23.98	1.07	1.4960(+1)	2.4234(−13)	481(−4)	60.2(+2)	531(+9)	74.8(+8)
140	74.98	23.02	2.00	1.4952(−3)	2.4186(−16)	490(+2)	57.7(+1)	539(+9)	66.0(0)
141	75.21	19.67	5.12	1.4934(−5)	2.4025(−1)	502(−2)	48.2(−2)	550(−5)	54.8(−9)
142	75.48	19.70	4.82	1.4935(0)	2.4022(+7)				
143	75.61	21.54	2.85	1.4949(+4)	2.4117(+20)	495(0)	54.5(+1)	539(0)	66.0(+2)
144	77.71	21.36	0.93	1.4926(−2)	2.4013(+31)				

[1] The figures in parentheses, when subtracted from the observed values, give computed values, or those obtained from figs. 1, 2, 3, and 4.

[2] The parenthetical figures in these two columns have been multiplied by 10^4.

III. CHANGES IN THE OBSERVED PROPERTIES OF THE GLASSES WITH CHANGES IN COMPOSITION

Correlation of the measured properties and composition of the glasses, as shown in figures 1 to 4, was based on the procedure used

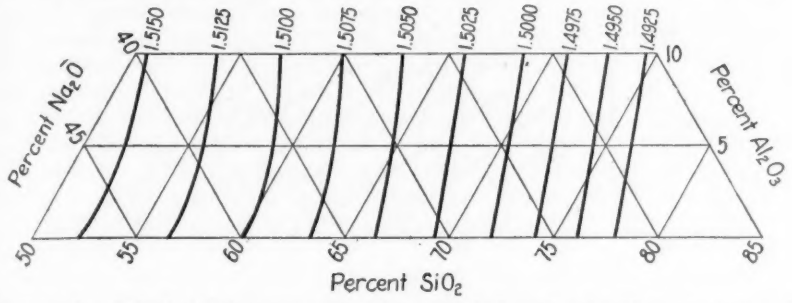

FIGURE 1.—*Relation between composition and index of refraction of some soda-silica-alumina glasses.*

for density-composition relations described in the following paragraph.[2]

The data were first plotted on a triordinate diagram according to their compositions. Then lines of constant density were obtained by first interpolating graphically between several pairs of neighboring points so chosen as to be near to a trial constant density value, and then sketching the lines through the interpolated points. The inter-

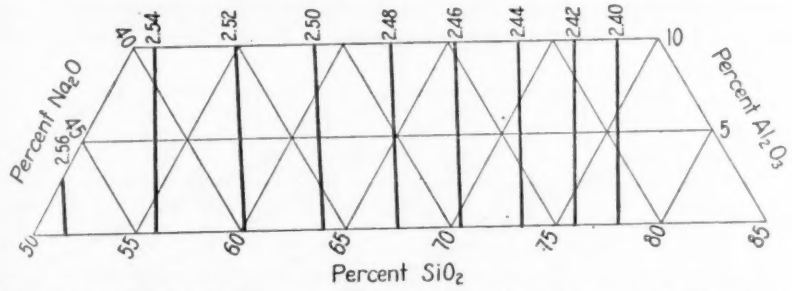

FIGURE 2.—*Relation between composition and density of some soda-silica-alumina glasses.*

sections of these lines with the silica-soda base line were established from the published data for that series of glasses.

The effects of changes in composition on index of refraction, density, critical temperature, and total expansion from room temperature to the critical temperature, and on softening temperature and the total expansion from room temperature to the softening temperature are indicated in figures 1, 2, 3, and 4, respectively.

[2] The following type of correlation was also worked out, but is not reported here. An algebraic solution, based on the previously reported factors for silica and soda, was used to determine factors for alumina, and from these factors constant density and refractivity lines were drawn on triordinate diagrams. This procedure necessarily yielded "break lines" (discussed in the report on density of the soda-lime glasses) running obliquely across the diagrams, dividing the field into four areas, corresponding to the number of silica-soda groups. Very little weight could be attached to the locations of these lines, however, owing partly to the limited amount of alumina present and partly to the fact that the total changes in the magnitude of the properties measured were relatively small.

The difference between the observed value of any of the properties of the alumina glasses and the value read from the appropriate figure is given parenthetically in table 1. Similar differences for the alumina-free glasses were presented in the reports previously cited.

It can be shown from a study of the graphs that additions of alumina to the soda-silica glasses should not significantly change the

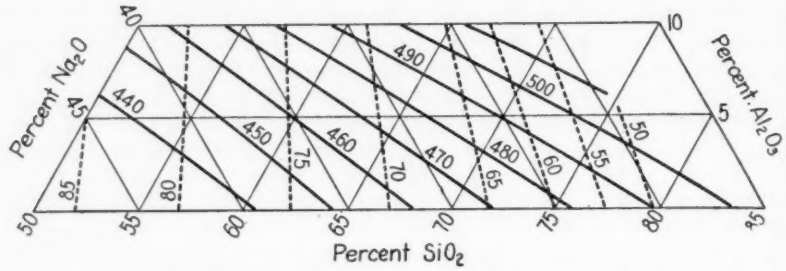

FIGURE 3.—*Relation between composition of some soda-silica-alumina glasses and critical temperatures (solid lines) and also linear expansions (broken lines) from 25° C to those temperatures.*

Temperatures are given in °C and expansions in μ/cm.

refractivity, density, or expansion to the softening point if, for the amount of alumina added, the silica and soda are reduced in the ratio 1:1.8, 1:1, or 1:1.5, respectively; the expansion to the critical temperatures changes, however, the ratio of substitution in the high-silica region being about 1:0.22 and in the low-silica region about 1:1.

In the report on expansion of the soda-lime-silica glasses equations were given for computing the expansions of these glasses in any range

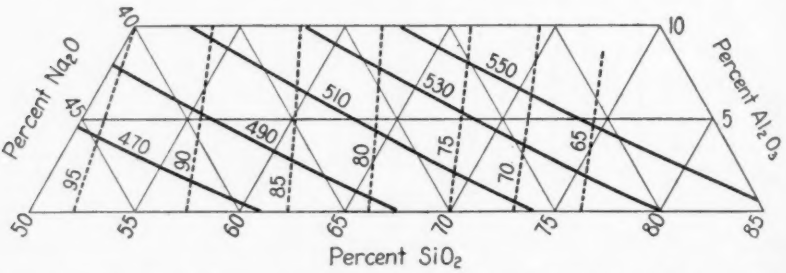

FIGURE 4.—*Relation between composition of some soda-silica-alumina glasses and softening temperatures (solid lines) and also linear expansions (broken lines) from 25° C to those temperatures.*

Temperatures are given in °C and expansions in μ/cm.

between 25 and 450° C. Following the procedure used for evaluating the constants in those equations, constants were computed from the data on the soda-alumina-silica glasses, with the following results:

$$E=aA+bB+dD$$
$$a=0.00043t-0.0000003t^2$$
$$b=0.00274t+0.0000035t^2$$
$$d=0.00001t-0.0000003t^2$$

in which E is the linear thermal expansion, in microns per centimeter, between 25° C and any temperature T less than 400° C, $t=T-25$,

A, B, and D are the percentages of silica, soda, and alumina, respectively, and a, b, and d are the indicated functions of temperatures. Although these constants were computed without respect to those previously reported,[3] it is noteworthy that the expansions for the soda-silica glasses computed from either set of constants for silica and soda are about the same. The average deviation (with approximately equal distribution between positive and negative values) of the calculated from the observed values of E at temperature intervals of 50° between 100 and 400° C, increased from 0.3 micron per centimeter at 100° C to 1.0 micron per centimeter at 400° C, the maximum individual deviation being 2.5 microns per centimeter.

Attention should be directed to the fact that the substitution of alumina for silica produces a very slight reduction in expansion to 400° C. This agrees with the conclusion reached by Turner and his coworkers[4] that the effect of alumina on expansion may be about the same as that of silica.

It may also be stated that, where comparisons can be made, the refractivity and density of the alumina glasses reported by us agree reasonably well with the results reported by Turner and his coworkers[5] if the small amounts of impurities in their glasses are taken into consideration.

WASHINGTON, August 27, 1934.

[3] The previously reported values are:

$$a = 0.00036t - 0.00000036t^2$$
$$b = 0.00245t + 0.0000046t^2$$

[4] S. English and W. E. S. Turner. *The thermal expansion of glasses containing aluminium.* J. Soc. Glass Tech. **5**, 183 (1921); and Violet Dimbleby, S. English, F. W. Hodkin, and W. E. S. Turner. *Further investigations of the influence of alumina on the properties of glass.* J. Soc. Glass Tech. **8**, 173 (1924).

[5] J. R. Clarke and W. E. S. Turner. *Note on some optical properties of the sodium-aluminium trisilicate glasses.* J. Soc. Glass Tech. **5**, 119 (1921); and S. English and W. E. S. Turner. *The density of glasses containing aluminium.* J. Soc. Glass Tech. **5**, 277 (1921).

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IMPACT AND STATIC TENSILE PROPERTIES OF BOLTS

By Herbert L. Whittemore, George W. Nusbaum, and Edgar O. Seaquist

ABSTRACT

This investigation was made to determine the properties of bolts under impact tensile loading and also under static tensile loading. 360 specimens were tested, representing all possible combinations of five different materials (chromium-nickel steel, cold-rolled steel, monel metal, bronze, and brass), four different bolt diameters (⅜, ½, ⅝, ¾ in.), and three different forms of screw threads (American National coarse, American National fine, and Dardelet). These threads are often used by engineers in this country for bolts. The U. S. Standard threads are almost the same as the American National coarse threads and the SAE threads almost the same as the American National fine threads. The bolts of different diameters were geometrically similar, the length between the head and the bearing face of the nut being five times the diameter, the thread extending inward from the face of the nut one diameter. In all cases the impact work for bolts with American National coarse threads was less than for bolts of the same size and material with American National fine threads. Except for the brass bolts and those cold-rolled steel bolts which showed brittle failures, the impact work for bolts with American National fine threads was approximately the same as for bolts of the same size and material with Dardelet threads. In all cases the impact work for bolts with Dardelet threads was much greater than for bolts of the same size and material having American National coarse threads.

Similar relations were observed for the static work and the maximum static load.

For bolts of the same size and having the same threads the bolt efficiencies were approximately the same for all of the materials.

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I. INTRODUCTION

Bolts are used in machines and structures to fasten together parts which cannot readily be made of one piece, or parts which may be removed for adjustment, cleaning, or repairs. In many cases these fastenings are subjected in service to static and to impact tensile loads. The designing engineer should know not only the greatest load which a bolt will sustain without failing but also the work (force times distance) required to rupture the bolt if, under unusual circumstances, the bolted structure should be subjected either to steady or to impact loads greater than the working loads for which the structure was designed.

The resistance to static loads depends upon the tensile strength of the bolt with nut. The resistance to impact loads, however, depends upon the work required to rupture the bolt under suddenly applied loads. This "impact work" depends upon the load-stretch curve under impact. This will be different from the load-stretch curve under static conditions and would be affected differently by the shape and surface of the threads and the character of the material.

Many investigations have shown that for metals under either tensile or transverse loading, the work required to rupture the specimen under loads applied slowly (the "static work") is not equal to the work required to rupture it under loads applied suddenly (the impact work). Similar differences between the static work and the impact work are to be expected for bolts.

A few investigators have studied the static and, sometimes, the impact strength of bolts. Their results were used as a guide when preparing the test program for this investigation. The effect of decreasing the cross-sectional area of the shank of the bolt to about the cross-sectional area at the bottom of the threads has been discussed by Kimball and Barr.[1] They point out that although decreasing the cross-sectional area lowers somewhat both the torsional and the tensile strength and the stiffness, it increases the impact work many times because it increases the stretch. They discuss decreasing the cross-sectional area by reducing the diameter of the shank, by longitudinal flutes and, preferably, by an axial hole. Drop tests upon bolts for Prof. Sweet's straight line engine showed that the impact work was increased about nine times by an axial hole.

The effect of differences in the pitch of the thread was studied by Major William R. King, U. S. Engineers.[2] His static tensile tests made at Watertown Arsenal on wrought-iron bolts, 1½ in. diameter having V threads, showed that the static work for bolts having 18 threads per inch was four times that for similar bolts having 6 threads per inch.

The shape of thread was considered by Langenberg.[3] His tensile impact tests at Watertown Arsenal on specimens having Acme threads (flat bottoms) showed greater stretch and greater impact work than similar specimens having a semicircular groove at the bottom of the thread.

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[1] Elements of Machine Design, p. 178 (1913).

[2] *Experiments with bolts and screw threads.* Trans. Am. Inst. Mining Engrs. **14, 90** (June 1885 and May 1886).

[3] *Investigation of failure of elevating screws on 1¼ in. D. C. no. 13 and 1¼ in. D. C. no. 14,* Tests of Metals p. 44 (1917). *Experimental data obtained on the Charpy impact machine.* Tests of Metals, p. 222 (1918). Also this latter paper by F. C. Langenberg. Bul. Am. Inst. Mining and Met. Engrs. no. 152, p. 1471 (Aug. 1919).

Beyer [4] made static tensile and tensile impact tests on cold-drawn steel bolts having Dardelet and U. S. Standard threads. He reported that bolts having Dardelet threads gave greater static tensile strength, greater stretch, and greater resistance to impact than similar bolts having U. S. Standard threads.

He found further [5] that the static tensile strength and the static work depended upon the length of the thread exposed between the head of the bolt and the face of the nut. As the length of exposed thread was decreased, the stretch under static load and also the static work decreased markedly being only one-half to one-third as great when no threads were exposed as when the exposure was two or three times the diameter of the bolt.

To keep this investigation within practicable limits it was decided to study the effect on the static work and impact work of differences in only three variables. They were the shape of the thread, the material, and the size of the bolt. The three shapes of thread which are used for most commercial bolts in this country were chosen. They were the American National coarse thread, the American National fine thread, and the Dardelet thread. The differences between the American National coarse thread and the U. S. Standard thread, and between the American National fine thread and the SAE thread are so small that there is no reason to believe that there would be appreciable differences in the static or the impact works. Therefore, the U. S. Standard thread and the SAE thread were not included. The results showed the effect of differences in the pitch for threads having the same profile (shape) as studied by King and the effect of differences in the profile of commercial threads as studied by Langenberg and Beyer.

The materials, chromium-nickel steel, cold-rolled steel, monel metal, bronze, and brass, were chosen as representative of the kinds of material which are much used for commercial bolts and nuts. It is believed that from the results on these five materials, the static work and the impact work for other materials can be estimated with sufficient accuracy for engineering purposes. If all the different materials used for bolts and nuts had been included the cost of this investigation would have been prohibitive.

Through the courtesy and cooperation of the Ordnance Department of the U. S. Army the impact tests were made in the large Charpy machine at Watertown Arsenal. This machine [6] having a nominal capacity of 2,170 ft-lb (300 kg-m) is the largest impact machine of the Charpy type in this country. Nisley gives the weight of the pendulum as 212.46 lb, the radius to the center of gravity as 5.34 ft, the maximum starting angle as 160°, the free return angle corresponding to the maximum starting angle as 158°, the velocity of impact (maximum starting angle), as 28.65 fps, the (actual) capacity as 2,203.2 ft-lb, the distance from the center of the specimen to the axis of rotation as 6.56 ft, the period of oscillation as 2.77 seconds, and the weight of the block or tup as 5.07 lb.

[4] Reports no. 2162 (Nov. 1929) and no. 2162A (Feb. 1930), Comparative Shock Resistance of Standard V Thread and Nut Connections and Dardelet Thread and Nut Connections. Columbia University (New York, N. Y.).

[5] Report no. 2207 (June 1930), Effect of Length of Thread Exposure upon the Static Tensile Strength and Energy to Rupture of Standard V and Dardelet Thread and Nut Connections. Columbia University (New York, N. Y.).

[6] Harold A. Nisley, *The relation between the dynamic and the static tensile tests.* Army Ordnance **4**, no. 20, 88-93 (Sept.-Oct. 1923).

For some of the materials it was estimated that specimens of the desired shape having a diameter greater than ¾ in. might not be ruptured in this impact machine. It was estimated, therefore, that in this machine bolts made from the strongest and most ductile material and having a diameter of ¾ in. could be tested and that the results obtained on bolts made from the weakest and least ductile material and having a diameter of ⅜ in. would be sufficiently accurate.

As bolts having a diameter less than ⅜ in. are seldom used under severe service conditions, the sizes chosen for the specimens were ⅜, ½, ⅝, and ¾ in. diameter.

To avoid the differences in work found by Beyer [7] caused by differences in the length of thread exposed between the head of the bolt

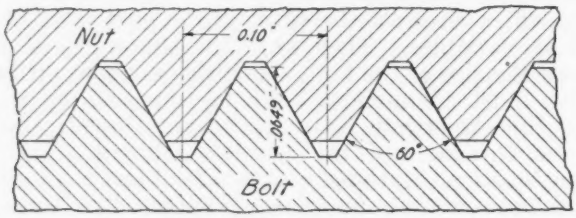

FIGURE 1.—*Section of the American National coarse thread for a bolt having a diameter of ¾ inch.*

and the face of the nut, all the tests were made with the thread exposure equal to one diameter of the bolt. It is believed that this is about the average thread exposure for commercial bolts under service conditions.

II. DESCRIPTION OF SCREW THREADS

The dimensions of the American National thread (coarse and fine) nuts and bolts are given in the Report of the National Screw Thread

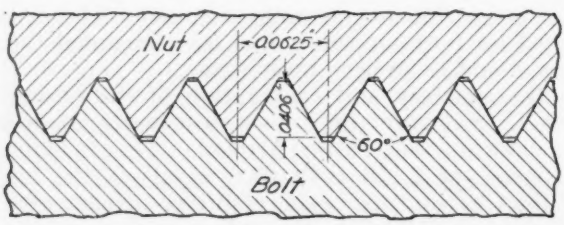

FIGURE 3.—*Section of the American National fine thread for a bolt having a diameter of ¾ inch.*

Commission, Misc. Pub. BS M89 (revised 1928). Those of the U. S. Standard thread are given in the Report of the Special Committee on a Uniform System of Screw Threads, Journal of the Franklin Institute, volume 49, pages 53 to 57 (1865) (adopted March 16, 1865, l. c. page 280).

The dimensional differences between the American National coarse thread and the U. S. Standard thread are so small that the bolts and nuts are usually interchangeable.

[7] Report no. 2207 (June 1930), Effect of Length of Thread Exposure upon the Static Tensile Strength and Energy to Rupture of Standard V and Dardelet Thread and Nut Connections. Columbia University, (New York, N. Y.).

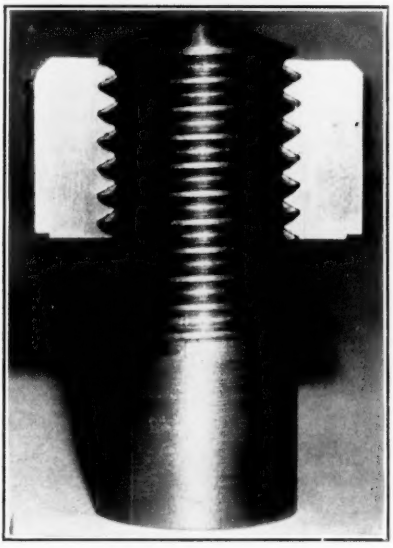

FIGURE 2.—*American National coarse thread, diameter of bolt ¾ inch.*

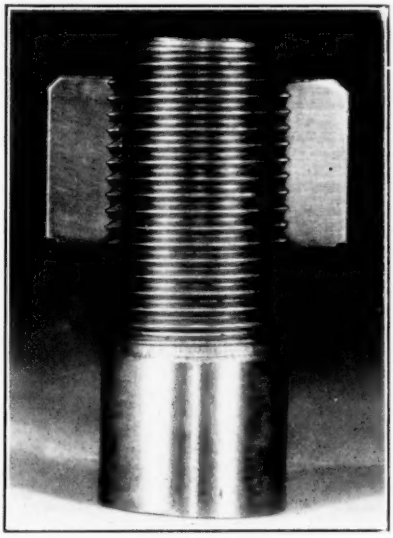

FIGURE 4.—*American National fine thread, diameter of bolt ¾ inch.*

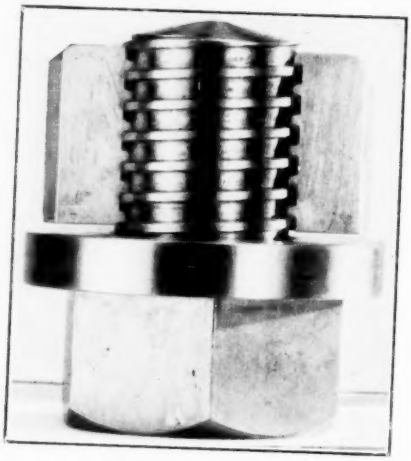

FIGURE 7.—*The Dardelet thread, diameter of bolt ¾ inch.*
Position of the nut when it rotates freely on the bolt.

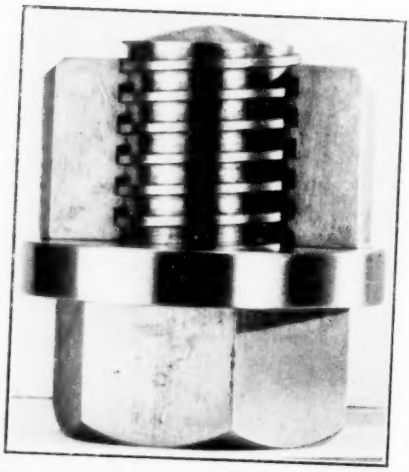

FIGURE 8.—*Dardelet thread, diameter of bolt ¾ inch.*
Position of the nut when thrust faces are in contact and the helical cones are elastically compressed, resisting unscrewing of the nut.

A drawing of the American National coarse thread for a ¾ in. bolt is shown in figure 1 and a photograph in figure 2. There are 10 threads per inch on this bolt. A drawing of the American National fine thread is shown in figure 3 for a ¾ in. bolt and a photograph in figure 4. There are 16 threads per inch on this bolt. For all diameters of bolt the pitch of the American National fine thread is less than that of the American National coarse thread.

Drawings of the Dardelet thread for a ¾ in. bolt are shown in figures 5 and 6, and photographs in figures 7 and 8. The inclination

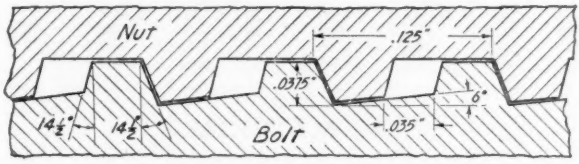

FIGURE 5.—*Section of the Dardelet thread for a bolt having a diameter of ¾ inch.*
Position of the nut when it rotates freely on the bolt.

of the sides or thrust faces of the threads is the same as that of the Acme thread, i. e., 14½° to the normal to the axis of the bolt. The surfaces at the major diameters are cylindrical with clearance between the nut and the bolt. The surfaces at the minor diameter of both the bolt and the nut are tapered, forming oblique helicoids with a generating angle of 6° to the axis of the bolt. For convenience these surfaces will, hereafter, be called "helical cones." When the nut is screwed on the bolt there is clearance between the threads, as shown in figures 5 and 7, and the nut rotates freely. When the nut

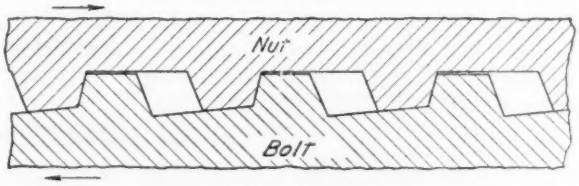

FIGURE 6.—*Section of the Dardelet thread for a bolt having a diameter of ¾ inch.*
Position of the nut when the thrust faces are in contact and the helical cones are elastically compressed resisting unscrewing of the nut.

is screwed down into contact with the parts to be clamped, the helical cones come into contact, and as the nut is tightened they are compressed elastically until the thrust faces of the threads come into contact as shown in figures 6 and 8. When the bolt is under the working tension most of the load is carried by the thrust faces. The elastic compression of the helical cones increases the frictional resistance of the nut to rotation.

The nominal dimensions of the threads and the number of threads for each size of specimen are given in table 1.

TABLE 1.—*Nominal dimensions of threads and number of threads*

Nominal size of specimen	Basic major diameter for all threads	American National coarse thread			American National fine thread			Dardelet thread		
		Number of threads per inch	Basic minor diameter	Ratio of diameters—minor: major	Number of threads per inch	Basic minor diameter	Ratio of diameters—minor: major	Number of threads per inch	Basic minor diameter	Ratio of diameters—minor: major
in.	in.		in.			in.			in.	
3/8	0.3750	16	0.2938	0.7835	24	0.3209	0.8557	12	0.3250	0.8667
1/2	.5000	13	.4001	.8002	20	.4350	.8700	10	.4400	.8800
5/8	.6250	11	.5069	.8110	18	.5528	.8845	8	.5500	.8800
3/4	.7500	10	.6201	.8268	16	.6688	.8917	8	.6750	.9000

III. MATERIAL

1. KINDS OF MATERIALS

The materials for the specimens were chromium-nickel steel (heat treated), cold-rolled steel, monel metal, bronze, and brass.

All nuts were made from hexagonal bars except those of chromium-nickel steel, which were made from round bars because hexagonal bars were unobtainable from stock. The diameter of these round bars was equal to the distance across flats of hexagonal nuts of the same nominal size. As the nuts were only hand tight when assembled for the tests no flats were required. The sizes of the bars are given in table 2.

TABLE 2.—*Size and Vickers numbers of material*

Purpose	Size		Number of bars	Vickers number			
	Diameter, if round; distance across flats, if hexagonal			Maximum	Minimum	Average	Load
	Round	Hexagonal					
Chromium-Nickel Steel							
	in.	in.					kg
Bolts	1 1/8		4	308	265	279	50
Nuts		5/8	1	291	289	290	50
		13/16	1	307	298	302	50
	1		1	291	287	289	50
	1 1/8		1	300	294	297	50
Cold-Rolled Steel							
Bolts	1 1/8		5	211	202	206	50
Nuts		5/8	1	254	251	252	50
		13/16	1	226	218	222	50
		1	1	217	217	217	50
		1 1/8	1	231	228	230	50
Monel Metal							
Bolts	1 1/8		4	277	254	262	50
Nuts		5/8	1	202	199	200	50
		13/16	1	202	199	200	50
		1	1	204	203	204	50
		1 1/8	1	204	189	196	50

TABLE 2.—*Size and Vickers numbers of material*—Continued

Purpose	Size: Diameter, if round; distance across flats, if hexagonal		Number of bars	Vickers number: Maximum	Minimum	Average	Load
	Round	Hexagonal					
	in.	in.					kg.
Bronze							
Bolts	1⅛		5	227	193	214	50
Nuts		⅝	1	183	179	181	50
		13/16	1	202	202	202	50
		1	1	212	209	210	50
		1⅛	1	201	190	196	50
Brass							
Bolts	1⅛		5	147	138	141	30
Nuts		⅝	1	162	158	160	30
		13/16	1	143	143	143	30
		1	1	159	158	158	30
		1⅛	1	138	138	138	30

2. IDENTIFICATION

An impression was made with a knurl on the surface of each bar for its entire length and the number of the bar stamped upon it every inch. As a portion of the original surface of the bar remained on each finished bolt and nut, the material was identified by the pattern of the knurling and the bar by the number.

3. VICKERS NUMBERS

Pieces ⅜ in. long were cut from both ends of each bar and the cut surface machined and polished. Using a Vickers machine and a diamond pyramid, three identations were made on the polished surface of each piece and the Vickers numbers determined. A load of 30 kg was used for brass and a load of 50 kg for all the other materials. The Vickers numbers are given in table 2.

4. CHEMICAL COMPOSITIONS

The results of chemical analyses of the material are given in table 3. The samples were taken from the bars having the nearest to the average Vickers number for the bolt stock and for the nut stock of each material.

TABLE 3.—*Chemical analyses of material*

Element	Chromium-nickel steel		Cold-rolled steel		Monel metal		Bronze		Brass	
	Bolt	Nut	Bolt	Nut	Bolt	Nut	Bolt	Nut	Bolt	Nut
	Per-cent	*Per-cent*	*Per-cent*	*Per-cent*	*Per-cent*	*Per-cent*	*Per-cent*	*Per-cent*	*Per-cent*	*Per-cent*
Carbon	0.37	0.33	0.12	0.18						
Manganese	.55	.57	.84	.92	1.3	1.2	0.9	1.2		
Phosphorus			.099	.115						
Sulphur			.12	.12						
Silicon			.01	.02			2.9	3		
Chromium	.52	.63								
Nickel	1.28	1.24			65.9	67.2		.1		0.02
Copper					31.2	29.8	95.9	95.1	61.7	62.4
Zinc [1]							.15	.39	34.89	34.36
Iron					1.7	1.8	.15	.21	.01	.02
Lead									3.3	3.1
Tin									.1	.1

[1] By difference.

The chromium-nickel steel was similar to SAE 3140 and had been heat treated to comply with ASTM A96–27, class C. The manufacturer stated that each lot was tested and was guaranteed to comply with this specification. The cold-rolled steel was described by the manufacturer as "cold finished screw stock" similar to SAE 1112. It was also similar to the Bessemer screw-steel grade in ASTM A108–30.

The bronze was a copper-silicon-manganese alloy similar to FS QQ–C–591. For convenience it is called "bronze" in this paper.

The brass was a free-cutting brass similar to grade B of FS QQ–C–611 and ASTM B16–29. For convenience it is called "brass" in this paper.

5. MECHANICAL PROPERTIES

The tensile properties of rolled bars are usually obtained on ASTM standard specimens, having a diameter of ½ in. and a gage length of 2 in. One specimen of this kind with threaded ends was taken from each bar of bolt material and one from each of the three bars of nut material which had the lowest, the nearest to the average, and the highest Vickers numbers. There was not sufficient nut material to take a specimen from each bar. The ultimate tensile strength, elongation, and reduction of area were determined.

Additional specimens were taken to determine accurately the proportional limit and the Young's modulus of elasticity. From each bolt and nut material one additional specimen was taken from the bar having the nearest to the average Vickers number and being, therefore, probably the most representative. For the bolt materials the additional specimen had a diameter of ½ in. and a gage length of 8 in. The ends were threaded. A Ewing extensometer (gage length 8 in.) was used on these specimens. As there was insufficient nut material for additional specimens having an 8-in. gage length, the additional specimens for the nut material were ASTM standard specimens of ½ in. diameter and 2 in. gage length.

Two Tuckerman optical strain gages[8] (gage length 2 in.), one on each side, were used on these specimens.

The proportional limit and the Young's modulus of elasticity for the additional specimens were determined from stress-strain diagrams. The proportional limit reported was the stress at which the stress-strain curve deviated from a straight line by a strain of 0.00001 in./in.

All the specimens were tested in a screw-power, beam and poise testing machine having a capacity of 50,000 lb. Calibration of this machine showed that the errors in the indicated load did not exceed 1 percent.

The mechanical properties of the materials are given in table 4.

[8] Proc. Am. Soc. Testing Materials, **23**, part 2, 602–610 (1923). Engineering (London) **116**, 222–223 (1923). World Eng. Congr. (Tokyo, Japan) Paper 335, p. 33–36.

TABLE 4.—*Mechanical properties of material*

Chromium-Nickel Steel

Purpose	Vickers number	Ultimate tensile strength	Elongation in 2 in.	Reduction of area	Proportional limit [1]	Young's modulus of elasticity	Purpose	Vickers number	Ultimate tensile strength	Elongation in 2 in.	Reduction of area	Proportional limit [1]	Young's modulus of elasticity
		lb/in.²	%	%	lb/in.²	lb/in.²			lb/in.²	%	%	lb/in.²	lb/in.²
Bolts	265	127,900	19.0	51.8			Nuts	287	127,600	17.0	63.4		
	266	126,200	19.0	62.9				298	130,800	15.0	59.7		
	269	128,700	20.5	61.7				307	136,100	16.0	59.2		
	308	140,800	13.5	58.9									
Av		130,900	18.0	58.8	[2] 25,000	[2] 28,500,000	Av		131,500	16.0	60.8	[3] 55,900	[3] 29,500,000

Cold-Rolled Steel

Bolts	202	85,600	17.5	48.9			Nuts	217	83,600	18.0	55.0		
	203	79,700	18.5	54.5				231	106,700	12.0	34.0		
	203	85,200	18.5	50.9				254	104,400	14.0	48.3		
	205	80,800	18.5	55.7									
	211	85,100	18.0	52.7									
Av		83,300	18.2	52.5	[2] 76,000	[2] 30,400,000	Av		98,200	14.7	45.8	[3] 66,000	[3] 29,800,000

Monel Metal

Bolts	254	104,200	20.5	54.7			Nuts	189	82,800	45.0	75.2		
	260	105,200	23.0	66.5				199	78,900	36.0	74.3		
	268	106,600	23.5	65.2				204	84,900	39.0	68.5		
	277	117,600	22.0	57.2									
Av		108,400	22.2	60.9	[2] 38,500	[2] 25,900,000	Av		82,200	40.0	72.7	[3] 21,200	[3] 25,000,000

Bronze

Bolts	193	80,800	24.0	60.7			Nuts	179	83,000	25.0	67.7		
	205	82,100	22.0	61.8				196	82,200	29.0	60.3		
	224	95,000	17.5	53.0				212	93,200	19.0	61.2		
	226	99,800	14.5	42.4									
	227	97,800	17.0	47.6									
Av		91,100	19.0	53.1	[2] 15,000	[2] 14,800,000	Av		86,100	24.3	63.1	[3] 23,800	[3] 13,600,000

Brass

Bolts	138	57,700	25.0	46.6			Nuts	138	58,400	27.0	45.7		
	138	58,800	24.5	45.4				143	58,800	21.0	49.0		
	141	57,600	25.5	46.2				162	68,100	12.0	37.8		
	141	60,700	22.0	37.5									
	147	58,700	22.0	37.2									
Av		58,700	23.8	42.6	[2] 20,000	[2] 13,500,000	Av		61,800	20.0	44.2	[3] 18,700	[3] 11,200,000

[1] Taken as the stress at which the stress-strain curve deviated from a straight line by a strain of 0.00001 in./in.
[2] Determined on 1 specimen, 8 in. gage length.
[3] Determined on 1 specimen, 2 in. gage length.

IV. SPECIMENS

1. MANUFACTURE

The bars were sent to a large manufacturer of bolts and nuts who made the bolt specimens. A drawing of the specimens is shown in figure 9. The threads on the larger end were cut with a die directly in the surface of the bar without any other machine work. The diameter d was the nominal size of the specimen. The portion hav-

ing the diameter d was machined and then threaded. Six similar specimens were made for each of the three types of thread of each of the five materials and for each of the four diameters (d), a total of 360 specimens.

The American National threads were ordered to meet the specifications of class 3, medium fit.[9] The American National threads, both coarse and fine, on the smaller ends of the specimens were cut by the manufacturer using a single point tool. The threads on the

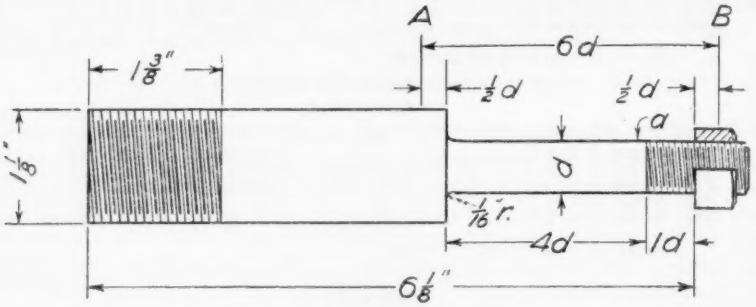

FIGURE 9.—*Specimen bolts whose nominal diameters* d *at the smaller ends were ⅜, ½, ⅝, and ¾ inch*

The dimensions given in inches were the same for all the bolt specimens. The other dimensions may be obtained by multiplying the coefficient by the nominal thread diameter, d. Thus the distance between A and B for the specimens having ½ in. threads was $6 \times ½ = 3$ in.

smaller ends of the Dardelet bolt specimens were cut by the Dardelet Threadlock Corporation on specimens finished, except for the threads, by the manufacturer of the bolt specimens. The nuts having Dardelet threads were made by this corporation from material furnished by the Bureau. The nuts having American National threads, coarse and fine, were made at the Bureau. The bolt and nut for each specimen were of the same kind of material.

2. MEASUREMENT OF SPECIMENS

The results of the measurements on the specimens are given in tables 5 to 9, inclusive.

[9] See Report of the National Screw Thread Commission, Misc. Pub. BS M141 (1933).

It was recognized that the strength of bolts depends considerably upon the minor diameter and that the minor diameter may not be within the permissible tolerances even though the pitch diameter is within the tolerances. Each bolt was measured, therefore, and both the minor diameter and the pitch diameter determined. For the minor diameter a Zeiss measuring microscope was used. In this instrument the microscope, having cross hairs in the field, was moved transversely by a screw micrometer having 100 divisions on the barrel of the micrometer. One rotation of the screw moved the microscope 1 mm. The difference between the micrometer reading when one of the cross hairs coincided with the root of the thread on one side of the bolt and then with the root on the other side, was taken as the minor diameter. The minor diameter in inches was computed from the metric value. It is believed that the error in these values did not exceed 0.0004 in. Values for the minimum minor diameter are not given in the Report of the National Screw Thread Commission; therefore these values for the American National bolts, both coarse and fine, were computed in accordance with footnote 1 on page 59 of the report. The differences between the actual minor diameter and the minimum minor diameter are given in the tables. A plus sign indicated that the actual diameter exceeded the minimum diameter and a minus sign that it was less. For the American National bolts the specimens for which this difference is minus did not comply with the requirements for minor diameter. For a few specimens the minor diameter exceeded the maximum allowable minor diameter. These values have been marked with a reference number. These specimens also did not comply with the requirements.

The dimensions of the Dardelet bolts were furnished by the Dardelet Threadlock Corporation. The differences for the minor diameter are the differences between the actual minor diameter and the nominal minor diameter because no tolerances were given.

No pitch diameter was given for the Dardelet threads; therefore, this diameter was not measured.

A screw micrometer having a suitable anvil and point was used for measuring the pitch diameter of the American National bolts. It is believed that the error in these readings did not exceed 0.001 in. The values which did not comply with the requirements as to pitch diameter after allowing for this possible error in the reading were marked with a reference number.

Neither the pitch of the bolt nor of the nuts was measured because it was believed that small differences in pitch had little effect upon the tensile strength of the specimens.

For many of the bolts, it was noticed that the axis of the threads on the larger end did not coincide with the axis of the bolt. Each bolt, therefore, was screwed into a threaded holder, on which was mounted a dial micrometer in such a position that rotation of the specimen in the holder indicated the amount of eccentricity of the body of the bolt near the small threads, at a, figure 9. The position of maximum eccentricity was marked on the bolt. The angle between the axis of the large thread and the axis of the bolt was computed from the readings of the dial. Angles of the order of 0.1 min of arc could be detected. For some of the bronze and the brass bolts this angle was too small to be measured, being less than this value.

TABLE 5.—*Results of measurements and tests; chromium-nickel steel*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
⅜ Inch American National Coarse																
Specimen no.	Bolt					Nut			Impact			Static				*Ratio* Impact work (col 12) / Static work (col 16)
	Bar no.	Actual minor diameter	Difference between actual and minimum minor diameter	Pitch diameter	Angle between axis of large thread and axis of bolt	Thickness	Obliquity of bearing face	Clearance between nut and bolt	Stretch, 5 diameters	Impact work: Each specimen	Impact work: Average	Stretch, 5 diameters	Maximum load	Static work: Each specimen	Static work: Average	
		in.	in.	in.	deg	in.	deg min	in.	in.	ft-lb	ft-lb	in.	lb	ft-lb	ft-lb	
1	2	0.2886	−0.0019	0.332	0.260	0.327	0 7	0.0042	0.12	90	94					1.21
2	1	.2832	−.0023	.331	.292	.328	0 7	.0035	.12	95						
3	4	.2858	−.0047	.331	.302	.327	0 13	.0050	.12	90						
4	1	.2941	+.0036	.333	.313	.324	1 12	.0030	.12	100						
5	4	.2862	−.0043	.331	.250	.327	0 52	.0040				0.10	9,370	76.4	77.4	
6	1	.2902	−.0003	.332	.339	.326	0 13	.0037				.10	9,520	78.4		
⅜ Inch American National Fine																
1	2	[1] 0.3244	+0.0060	0.348	0.271	0.328	0 13	0.0036	0.13	128	132					1.11
2	3			.347	.281	.326	0 7	.0044		[2] 128						
3	3	.3216	+.0032	.347	.026	.327	0 7	.0043	.14	134						
4	3	.3224	+.0040	.348	.391	.329	0 7	.0042	.16	134						
5	3	.3193	+.0009	.348	.287	.326	0 7	.0045				0.15	10,810	121	119	
6	2	.3224	+.0040	.348	.391	.326	0 7	.0045				.13	12,100	117		

3/8 Inch Dardelet

3	2	0.3271	+0.0021		0.063	0.502	0	7	0.0005	0.14	128	133					1.15
4	4	.3251	+.0001		.130	.502	0	20	.0020	.15	134						
5	4	.3267	+.0017		.104	.500	0	13	.0004	.17	140						
6	1	.3255	+.0005		.115	.503	0	26	.0020	.14	128						
1	3	.3251	+.0001		.130	.502	0	13	.0014				0.14	10,960	117	116	
2	4	.3267	+.0017		.099	.502	0	39	.0008				.13	11,130	115		

½ Inch American National Coarse

1	4	0.3917	−0.0046	0.447	0.554	0.437	0	5	0.0042	0.16	229	231					1.13
2	3	.3945	−.0018	.447	.309	.436	0	5	.0042	.15	236						
3	3	.3925	−.0038	.447	.394	.434	0	0	.0039	.16	229						
4	3	.3949	−.0014	.447	.389	.437	0	5	.0044	.15	229						
5	3	.4043	+.0080	.450	.384	.432	0	10	.0048				0.16	17,800	215	204	
6	3	.3964	+.0001	.447	.251	.436	0	5	.0035				.14	17,720	193		

½ Inch American National Fine

1	1	0.4354	+0.0030	0.465	0.357	0.438	0	5	0.0015	0.20	330	325					1.08
2	2	.4338	+.0014	.465	.352	.439	0	5	.0012	.16	316						
3	2	.4346	+.0022	.466	.330	.436	0	10	.0012	.17	323						
4	1	.4335	+.0011	.466	.357	.439	0	5	.0009	.19	330						
5	4	.4386	+.0062	.465	.309	.436			.0030				0.20	20,050	306	301	
6	3	.4354	+.0030	.465	.357	.436			.0012				.19	20,110	296		

½ Inch Dardelet

1	1	0.4382	−0.0018		0.128	0.627	0	14	0.0003	0.19	330	338					1.40
2	4	.4394	−.0006		.112	.624	0	14	.0010	.20	338						
3	4	.4390	−.0010		.160	.626	0	5	.0005	.20	346						
4	1	.4390	−.0010		.117	.625	0	14	.0007	.20	338						
5	1	.4390	−.0010		.112	.628	0	5	.0008				0.15	20,630	263	241	
6	2	.4390	−.0010		.096	.624	0	19	.0014				.12	22,150	219		

[1] Did not comply with requirements.

[2] Thread stripped—not included in the average.

TABLE 5.—*Results of measurements and tests; chromium-nickel steel*—Continued

1	2	3	4	5	6	7	8		9	10	11	12	13	14	15	16	17
⅝ Inch American National Coarse																	
Specimen no.	Bolt					Nut			Clearance between nut and bolt	Impact			Static				*Ratio* Impact work (col 12) / Static work (col 16)
	Bar no.	Actual minor diameter	Difference between actual and minimum minor diameter	Pitch diameter	Angle between axis of large thread and axis of bolt	Thickness	Obliquity of bearing face			Stretch, 5 diameters	Impact work: Each specimen	Impact work: Average	Stretch, 5 diameters	Maximum load	Static work: Each specimen	Static work: Average	
		in.	in.	in.	deg	in.	deg	min	in.	in.	ft-lb	ft-lb	in.	lb	ft-lb	ft-lb	
1	4	0.5035	+0.0007	0.562	0.371	0.548	0	8	0.0055	0.22	485	451					1.18
2	3	.5016	−.0012	.562	.497	.543	0	4	.0044	.19	466						
3	2	.5039	+.0011	.563	.344	.546	0	4	.0057	.15	426						
4	2	.5020	−.0008	.562	.382	.548	0	0	.0051	.15	426						
5	3	.5079	+.0051	.562	.344	.546	0	0	.0056				0.17	28,850	382	383	
6	1	.5035	+.0007	.562	.289	.544	0	4	.0049				.17	28,710	384		
⅝ Inch American National Fine																	
1	3			0.586	0.229	0.545	0	15	0.0043		[2] 362	695					1.12
2	3	0.5500	+0.0002	.586	.284	.546	0	4	.0047	0.24	683						
3	4	.5500	+.0002	.586	.459	.546	0	4	.0053	.25	683						
4	3	[1] .5579	+.0019	.587	.491	.546	0	0	.0046	.25	720						
5	3	.5512	+.0014	.587	.349	.545	0	19	.0046				0.25	32,320	622	623	
6	1	.5516	+.0018	.588	.448	.547	0	4	.0058				.25	32,670	624		
⅝ Inch Dardelet																	
1	3	0.5512	+0.0012		0.131	0.749	0	15	0.0023	0.25	609	605					1.26
2	3	.5504	+.0004		.115	.750	0	15	.0028	.26	637						
3	2	.5500	.0000		.087	.750	0	11	.0016	.21	555						
4	4	.5502	+.0002		.109	.749	0	11	.0013	.25	618						
5	2	.5498	−.0002		.126	.752	0	8	.0010				0.17	35,000	496	482	
6	2	.5507	+.0007		.115	.751	0	8	.0012				.16	36,070	469		

¾ Inch American National Coarse

¾ Inch American National Coarse

1	4	0.6106	−0.0049	0.682	0.274	0.656	0	0	0.0046	0.23	895	883				1.24
2		.6118	−.0037	.681	.425	.656	0	0	.0060	.25	895					
3	2	.6142	−.0013	.683	.302	.654	0	3	.0015	.22	885					
4		.6106	−.0049	.681	.302	.657	0	3	.0049	.24	856					
5	4	.6161	+.0006	.683	.268	.653	0	3	.0033				0.21	42,800	723	714
6	3	.6118	−.0037	.680	.402	.657	0	0	.0060				.21	41,610	705	

¾ Inch American National Fine

1	4	0.6709	+0.0053	0.709	0.369	0.656	0	7	0.0058	0.31	1,298	1,241				1.21
2	2	.6665	+.0009	.706	.582	.650	0	3	.0033	.29	1,201					
3	2	.6705	+.0049	.707	.268	.653	0	3	.0035	.27	1,194					
4	4	.6713	+.0057	.708	.632	.658	0	0	.0040	.31	1,270					
5	2	.6705	+.0049	.707	.291	.657	0	0	.0052				0.22	53,500	923	1,026
6	1	.6720	+.0064	.708	.408	.658	0	0	.0035				.30	48,310	1,130	

¾ Inch Dardelet

1	2	0.6775	+0.0025		0.095	0.874	0	10	0.0006	0.26	1,132	1,216				1.26
2	4	.6772	+.0022		.123	.874	0	7	.0005	.30	1,260					
3	1	.6762	+.0012		.106	.873	0	13	.0019	.32	1,240					
4		.6772	+.0022		.123	.874	0	17	.0015	.31	1,231					
5	4	.6756	+.0006		.117	.873	0	13	.0007				0.25	47,800	998	962
6		.6750	.0000			.882							.24	47,930	926	

[1] Did not comply with requirements.

[2] Thread stripped—not included in the average.

TABLE 6.—*Results of measurements and tests: cold-rolled steel*

1	2	3	4	5	6	7	8		9	10	11	12	13	14	15	16	17
⅜ Inch American National Coarse																	
		Bolt				Nut				Impact			Static				
Specimen no.	Bar no.	Actual minor diameter	Difference between actual and minimum minor diameter	Pitch diameter	Angle between axis of large thread and axis of bolt	Thickness	Obliquity of bearing face		Clearance between nut and bolt	Stretch, 5 diameters	Impact work		Stretch, 5 diameters	Maximum load	Static work		*Ratio* Impact work (col. 12) / Static work (col. 16)
											Each specimen	Average			Each specimen	Average	
		in.	in.	in.	deg	in.	deg	min	in.	in.	ft-lb	ft-lb	in.	lb	ft-lb	ft-lb	
1	4	0.2862	−0.0043	0.332	0.099	0.327	0	7	0.0051	0.10	70	68					1.34
2	4	.2858	−.0047	.332	.271	.327	0	7	.0053	.10	64						
3	2	.2866	−.0039	.332	.240	.327	0	0	.0046	.11	70						
4	5	.2866	−.0039	.332	.177	.326	0	7	.0053	.11	70						
5	5	.2866	−.0039	.331	.208	.326	0	0	.0060				0.10	6,370	50.9	50.8	
6	2	.2854	−.0051	.331	.438	.325	0	7	.0059				.10	6,630	50.8		
⅜ Inch American National Fine																	
1	2	0.3205	+0.0021	0.348	0.073	0.325	0	7	0.0031	0.13	90	92					1.22
2	3	.3201	+.0017	.348	.182	.328	0	0	.0031	.14	95						
3	5	.3193	+.0009	.348	.156	.322	0	0	.0039	.14	95						
4	1	.3205	+.0021	.348	.276	.323	0	7	.0040	.13	90						
5	5	.3197	+.0013	.348	.250	.323	0	0	.0028				0.13	7,520	75.2	75.4	
6	3	.3197	+.0013	.348	.255	.322	0	7	.0039				.13	7,250	75.5		

⅜ Inch Dardelet

1	1	0.3251	+0.0001		0.109	0.500	0	33	0.0012	0.14	95	95					1.17
2	1	.3255	+.0005		.161	.498	0	26	.0005	.18	100						
2	3	.3263	+.0013		.104	.501	0	33	.0008	.16	95						
4	1	.3255	+.0005		.083	.499	0	26	.0004	.15	90						
5	2	.3255	+.0005		.099	.499	0	33	.0007				0.13	7,920	82.4	81.4	
6	4	.3255	+.0005		.156	.496	0	26	.0008				.14	7,210	80.5		

½ Inch American National Coarse

1	2	0.3921	−0.0042	0.447	0.304	0.438	0	5	0.0055	0.03	[2] 50	164					1.39
2	4	.3976	+.0013	[1] .452	.245	.437	0	5	.0015	.15	164						
3	2	.3964	+.0001	[1] .451	.373	.437	0	0	.0018	.04	[2] 60						
4	3	.3898	−.0065	.447	.261	.438	0	10	.0060	.03	[2] 50						
5	1	.3976	+.0013	[1] .451	.250	.439	0	14	.0013				0.12	12,360	116	118	
6	3	.3984	+.0021	[1] .452	.426	.436	0	5	.0003				.13	11,690	121		

½ Inch American National Fine

1	3	0.4346	+0.0022	0.466	0.240	0.439	0	5	0.0044	0.19	229	226					1.26
2	1	.4303	−.0021	.466	.293	.429	0	5	.0009	.17	216						
3	4	.4287	−.0037	.466	.224	.437	0	5	.0012	.19	223						
4	5	.4342	+.0018	.466	.261	.433	0	5	.0011	.18	236						
5	3	.4346	+.0022	.466	.272	.433	0	10	.0011				0.17	13,300	182	180	
6	4	.4338	+.0014	.466	.298	.433	0	5	.0012				.18	13,020	178		

½ Inch Dardelet

3	5	0.4386	−0.0014		0.101	0.627	0	14	0.0028	0.20	236	243					1.34
4	1	.4390	−.0010		.091	.626	0	19	.0014	.21	250						
5	5	.4377	−.0023		.080	.627	0	10	.0016	.21	250						
6	1	.4402	+.0002		.101	.628	0	10	.0011	.20	236						
1	2	.4386	−.0014		.101	.626	0	10	.0008				0.16	14,150	187	181	
2	4	.4386	−.0014		.117	.627	0	14	.0030				.17	13,030	175		

[1] Did not comply with requirements.

[2] Brittle failure—not included in the average.

TABLE 6.—*Results of measurements and tests: cold-rolled steel*—Continued

1	2	3	4	5	6	7	8		9	10	11	12	13	14	15	16	17
⅝ Inch American National Coarse																	
Specimen no.	Bolt					Nut			Clearance between nut and bolt	Impact			Static				*Ratio* Impact work (col. 12) / Static work (col. 16)
	Bar no.	Actual minor diameter	Difference between actual and minimum minor diameter	Pitch diameter	Angle between axis of large thread and axis of bolt	Thickness	Obliquity of bearing face			Stretch, 5 diameters	Impact work		Stretch, 5 diameters	Maximum load	Static work		
											Each specimen	Average			Each specimen	Average	
		in.	in.	in.	deg	in.	deg	min	in.	in.	ft-lb	ft-lb	in.	lb	ft-lb	ft-lb	
1	1	0.5047	+0.0019	0.562	0.311	0.547	0	4	0.0053	0.20	323	331					1.35
2	5	.5047	+.0019	.562	.388	.543	0	4	.0054	.16	316						
3	5	.5043	+.0015	.562	.448	.541	0	0	.0060	.20	346						
4	1	.5118	+.0090	.562	.388	.545	0	4	.0060	.19	338						
5	1	.5035	+.0007	.562	.464	.546	0	4	.0055				0.16	18,290	231	244	
6	3	.5051	+.0023	.562	.229	.546	0	4	.0057				.17	18,680	257		
⅝ Inch American National Fine																	
1	4	0.5492	−0.0006	0.586	0.295	0.546	0	0	0.0052	0.26	434	445					1.28
2	3	.5492	−.0006	.587	.448	.546	0	4	.0050	.24	451						
3	5	.5481	−.0017	.586	.055	.545	0	8	.0050	.24	451						
4	2	.5523	+.0025	.587	.229	.545	0	8	.0049	.03	[2]112						
5	3	.5484	−.0014	.587	.311	.546	0	4	.0048				0.21	21,260	355	348	
6	2	.5520	−.0022	.587	.229	.546	0	8	.0054				.19	22,410	342		

⅝ Inch Dardelet

1	4	0.5498	−0.0002		0.142	0.750	0	4	0.0027	0.25	418	435					1.32
2	4	.5507	+.0007		.131	.752	0	8	.0016	.26	460						
3	2	.5500	.0000		.071	.751	0	8	.0026	.23	418						
4	1	.5512	+.0012		.098	.753	0	11	.0012	.25	443						
5	2	.5512	+.0012		.082	.749	0	11	.0024								
6	5	.5500	.0000		.126	.749	0	15	.0005				0.19	22,020	343	330	
													.17	22,000	317		

¾ Inch American National Coarse

1	4	0.6121	−0.0034	0.682	0.456	0.653	0	3	0.0042	0.25	564	564					1.31
2	1	.6114	−.0041	.682	.497	.655	0	3	.0049	.03	[2] 90						
3	5	.6122	−.0033	.681	.520	.655	0	3	.0057	.03	[2] 106						
4	2	.6088	−.0067	.681	.274	.655	0	0	.0052	.03	[2] 79						
5	1	.6118	−.0037	.681	.235	.652	0	0	.0043				0.19	28,860	438	432	
6	1	.6118	−.0037	.682	.553	.652	0	3	.0053				.18	28,800	426		

¾ Inch American National Fine

1	2	0.6630	−0.0026	0.707	0.212	0.656	0	7	0.0038	0.14	[2] 418	768					1.25
2		.6643	−.0013	.708	.514	.650	0	0	.0037	.27	768						
3	4	.6634	−.0022	.707	.553	.655	0	3	.0042	.10	[2] 278						
4	1	.6639	−.0017	.707	.693	.656	0	7	.0045	.05	[2] 128						
5	3	.6642	−.0014	.707	.358	.650	0	3	.0041				0.25	31,420	611	616	
6	3	.6630	−.0026	.708	.564	.655	0	3	.0045				.26	31,270	620		

¾ Inch Dardelet

1	5	0.6764	+0.0014		0.095	0.876	0	10	0.0013	0.30	856	834					1.39
2	4	.6764	+.0014		.145	.874	0	7	.0015	.30	817						
3	3	.6752	+.0002		.134	.875	0	7	.0020	.33	836						
4		.6762	+.0012		.134	.874	0	10	.0012	.30	826						
5	3	.6740	−.0010		.123	.875	0	10	.0017				0.24	31,800	618	599	
6	3	.6756	+.0006			.878							.22	31,720	580		

[2] Brittle failure—not included in the average.

TABLE 7.—*Results of measurements and tests: monel metal*

1	2	3	4	5	6	7	8		9	10	11	12	13	14	15	16	17
⅜ Inch American National Coarse																	
Specimen no.	Bolt					Nut			Clearance between nut and bolt	Impact			Static				*Ratio* Impact work (col. 12) / Static work (col. 16)
	Bar no.	Actual minor diameter	Difference between actual and minimum minor diameter	Pitch diameter	Angle between axis of large thread and axis of bolt	Thickness	Obliquity of bearing face			Stretch, 5 diameters	Impact work		Stretch, 5 diameters	Maximum load	Static work		
											Each specimen	Average			Each specimen	Average	
		in.	in.	in.	deg	in.	deg	min	in.	in.	ft-lb	ft-lb	in.	lb	ft-lb	ft-lb	
1	2	0.2874	−0.0031	0.332	0.354	0.323	0	13	0.0018	0.14	100	100					1.22
2	3	.2874	−.0031	.333	.313	.328	0	13	.0021	.14	100						
3	3	.2886	−.0019	.332	.313	.326	0	7	.0013	.14	100						
4	3	.2870	−.0035	.332	.386	.333	0	39	.0011	.14	100						
5	3	.2862	−.0043	.332	.469	.336	0	52	.0012				0.13	8,300	83.2	82.2	
6	3	.2882	−.0023	.332	.344	.326	0	39	.0015				.12	8,370	81.2		
⅜ Inch American National Fine																	
1	2	0.3193	+0.0009	0.348	0.271	0.340	0	20	0.0030	0.15	134	131					1.08
2	3	.3185	+.0001	.348	.260	.340	0	20	.0040	.15	128						
3	3	.3185	+.0006	.348	.250	.327	0	0	.0037	.16	134						
4	3	.3201	+.0017	.347	.292	.328	0	26	.0035	.15	128						
5	3	.3201	+.0017	.348	.302	.338	0	20	.0036				0.16	9,650	117	121	
6	2	.3212	+.0028	.348	.167	.326	0	39	.0033				.15	10,600	125		

⅜ Inch Dardelet

1	3	.3267	+0.0017		0.135	0.503	0	26	0.0004	0.16	128	131					1.08
2	2	.3251	+.0001		.120	.501	0	13	.0028	.14	128						
3	2	.3267	−.0017		.115	.501	0	13	.0005	.16	134						
4	2	.3271	+.0021		.125	.500	0	13	.0006	.15	134						
5	2	.3263	+.0013		.083	.503	0	13	.0006				0.14	10,660	122	122	
6	3	.3260	+.0010			.512							.15	9,680	121		

½ Inch American National Coarse

1	2	0.3909	−0.0054	0.448	0.256	0.437	0	5	0.0070	0.17	250	245					1.18
2	2	.3909	−.0054	.448	.320	.443	0	24	.0030	.16	236						
3	2	.3921	−.0042	.448	.245	.436	0	5	.0072	.17	243						
4	2	.3949	−.0014	.448	.288	.438	0	5	.0073	.17	250						
5	3	.3913	−.0050	.448	.224	.438	0	10	.0073				0.18	15,260	216	208	
6	2	.3898	−.0065	.448	.234	.440	0	39	.0033				.16	16,070	199		

½ Inch American National Fine

1	2	0.4331	+0.0007	0.466	0.192	0.437	0	19	0.0023	0.22	338	344					1.23
2	2	.4311	−.0013	.466	.309	.437	0	24	.0023	.21	338						
3	3	.4342	+.0018	.466	.277	.438	0	34	.0015	.23	346						
4	2	.4323	−.0001	.466	.299	.437	0	24	.0015	.22	354						
5	2	.4319	−.0005	[1] .464	.213	.437	0	14	.0035				0.18	18,190	268	280	
6	2	.4354	+.0030	.465	.352	.435	0	29	.0035				.19	18,870	292		

½ Inch Dardelet

3	2	0.4377	−0.0023		0.133	0.629	0	10	0.0025	0.21	323	317					1.20
4	2	.4382	−.0018		.107	.628	0	19	.0023	.20	293						
5	3	.4386	−.0014		.160	.623	0	10	.0016	.24	330						
6	3	.4377	−.0023		.123	.626	0	14	.0013	.23	323						
1	2	.4382	−.0018		.091	.623	0	19	.0026				0.17	19,180	271	264	
2	2	.4390	−.0010		.133	.627	0	19	.0023				.17	18,320	258		

[1] Did not comply with requirements.

TABLE 7.—*Results of measurements and tests: monel metal*—Continued

1	2	3	4	5	6	7	8		9	10	11	12	13	14	15	16	17
⅜ Inch American National Coarse																	
Specimen no.	Bolt					Nut			Clearance between nut and bolt	Impact			Static				*Ratio* Impact work (col. 12) / Static work (col. 16)
	Bar no.	Actual minor diameter	Difference between actual and minimum minor diameter	Pitch diameter	Angle between axis of large thread and axis of bolt	Thickness	Obliquity of bearing face			Stretch, 5 diameters	Impact work		Stretch, 5 diameters	Maximum load	Static work		
											Each specimen	Average			Each specimen	Average	
		in.	in.	in.	deg	in.	deg	min	in.	in.	ft-lb	ft-lb	in.	lb	ft-lb	ft-lb	
1	4	0.4941	−0.0087	[1] 0.561	0.196	0.548	0	0	0.0068	0.21	451	453					1.21
2	3	.4941	−.0087	.564	.218	.549	0	15	.0041	.21	443						
3	4	.4917	−.0111	.562	.306	.548	0	8	.0083	.24	466						
4	4	.4921	−.0107	.562	.240	.551	0	4	.0073	.21	451						
5	3	.4941	−.0087	.562	.240	.548	0	8	.0058				0.18	24,580	364	374	
6	1	.4917	−.0111	.562	.218	.547	0	23	.0052				.21	23,650	384		
⅝ Inch American National Fine																	
1	2	0.5476	−0.0022	0.588	0.153	0.545	0	8	0.0033	0.26	628	684					1.26
2	4	.5488	−.0010	.586	.273	.547	0	8	.0035	.27	697						
3	1	.5488	−.0010	.588	.295	.548	0	4	.0044	.29	711						
4	4	.5480	−.0018	.587	.251	.547	0	8	.0029	.28	702						
5	1	.5476	−.0022	.588	.350	.545	0	4	.0048				0.25	27,690	526	542	
6	1	.5472	−.0026	.587	.262	.540	0	8	.0038				.26	27,520	558		

⅝ Inch Dardelet

1	1	0.5507	+0.0007		0.044	0.749	0	8	0.0018	0.27	637	637					1.23
2	4	.5507	+.0007		.136	.752	0	19	.0015	.28	637						
3	4	.5512	+.0012		.066	.752	0	15	.0009	.28	628						
4	1	.5504	+.0004		.055	.747	0	27	.0010	.28	646						
5	4	.5512	+.0012		.147	.750	0	8	.0013				0.23	28,160	528	517	
6	4	.5500	.0000		.098	.749	0	15	.0014				.22	27,900	506		

¾ Inch American National Coarse

2	1	0.6071	−0.0084	0.682	0.358	0.658	0	3	0.0085	0.27	875	880					1.23
3	1	.6051	−.0104	.682	.347	.658	0	3	.0082	.29	856						
5	4	.6039	−.0116	.682	.246	.656	0	3	.0075	.29	895						
6	1	.6067	−.0088	.682	.140	.657	0	3	.0073	.28	895						
1	1	.6051	−.0104	.682	.212	.655	0	3	.0070				0.26	35,480	718	716	
4	1	.6035	−.0120	.682	.380	.656	0	0	.0078				.26	35,080	714		

¾ Inch American National Fine

1	1	0.6630	−0.0026	0.708	0.391	0.659	0	3	0.0034	0.35	1,250	1,262					1.29
2	4	.6618	−.0038	.708	.324	.657	0	3	.0037	.35	1,288						
5		.6622	−.0034	.708	.402	.659	0	3	.0022	.34	1,231						
6	4	.6661	+.0005	.708	.212	.657	0	3	.0029	.34	1,280						
3	1	.6634	−.0022	.708	.134	.658	0	3	.0020				0.31	40,330	977	976	
4		.6602	−.0054	.708	.414	.653	0	0	.0019				.31	40,270	976		

¾ Inch Dardelet

1	1	0.6762	+0.0012		0.101	0.878	0	13	0.0011	0.34	1,211	1,221					1.17
2		.6728	−.0022		.151	.877	0	10	.0014	.36	1,240						
3	1	.6756	+.0006		.196	.875	0	20	.0006	.35	1,221						
4	1	.6756	+.0006		.095	.865	0	10	.0027	.35	1,211						
5	1	.6752	+.0002		.151	.869	0	13	.0026				0.33	41,120	1,060	1,045	
6	1	.6756	+.0006		.123	.880	0	10	.0019				.32	41,450	1,030		

[1] Did not comply with requirements.

TABLE 8.—*Results of measurements and tests: bronze*

1	2	3	4	5	6	7	8		9	10	11	12	13	14	15	16	17
¾ Inch American National Coarse																	
Specimen no.	Bolt					Nut			Clearance between nut and bolt	Impact			Static				*Ratio* Impact work (col. 12) / Static work (col. 16)
	Bar no.	Actual minor diameter	Difference between actual and minimum minor diameter	Pitch diameter	Angle between axis of large thread and axis of bolt	Thickness	Obliquity of bearing face			Stretch, 5 diameters	Impact work: Each specimen	Impact work: Average	Stretch, 5 diameters	Maximum load	Static work: Each specimen	Static work: Average	
		in.	in.	in.	deg	in.	deg	min	in.	in.	ft-lb	ft-lb	in.	lb	ft-lb	ft-lb	
1	1	0.2866	−0.0039	0.332	0.031	0.325	0	7	0.0039	0.13	70	62					1.22
2	1	.2870	−.0035	.331	.036	.321	0	7	.0057	.12	50						
4	2	.2862	−.0043	[1].330	.031	.322	0	7	.0053	.10	70						
5	2	.2870	−.0035	.331	.068	.325	0	7	.0048	.08	60						
3	2	.2862	−.0043	.331	.083	.329	0	7	.0045				0.10	7,230	58.8	50.8	
6	2	.2838	−.0067	[1].329	.047	.326	0	7	.0056				.07	6,930	42.8		
⅜ Inch American National Fine																	
1	1	0.3201	+0.0017	0.347	0.057	0.330	0	7	0.0034	0.14	90	92					1.25
2	1	.3193	+.0009	.347	.026	.329	0	13	.0027	.15	90						
3	2	[1].3252	+.0078	.347	.016	.328	0	7	.0031	.13	90						
4	1	.3236	+.0052	.347	.031	.329	0	7	.0033	.17	100						
5	2	.3201	+.0017	[1].351	.016	.329	0	7	.0008				0.10	8,410	73.4	73.8	
6	1	.3193	+.0009	.347	.031	.330	0	0	.0035				.12	7,290	74.2		

3/8 Inch Dardelet

1	1	0.3260	+0.0010		0.182	0.503	0	46	0.0007	0.18	100	98					1.25
2	1	.3263	+.0013		.083	.500	0	13	.0008	.14	95						
3	1	.3260	+.0010		.182	.502	0	59	.0009	.18	100						
4	2	.3248	+.0002		.156	.501	0	33	.0019	.12	95						
5	2	.3263	+.0013		.172	.501	0	33	.0012				0.11	8,430	82.6	78.4	
6	3	.3248	−.0002		.130	.501	0	20	.0021				.10	8,310	74.2		

1/2 Inch American National Coarse

1	1	0.3906	−0.0057	0.447	0.000	0.433	0	5	0.0049	0.17	177	177					1.32
2	4	.3909	−.0054	.446	.021	.433	0	5	.0046	.19	183						
3	2	.3957	−.0006	.447	.000	.435	0	5	.0045	.16	177						
4	4	.3886	−.0077	.446	.000	.437	0	5	.0057	.17	170						
5	2	.3921	−.0042	.446	.000	.438	0	0	.0053				0.13	12,720	136	134	
6	1	.3886	−.0077	.446	.021	.435	0	5	.0054				.14	11,360	131		

1/2 Inch American National Fine

1	1	0.4280	−0.0044	[1] 0.464	0.000	0.435	0	5	0.0020	0.24	250	241					1.37
2	1	.4272	−.0052	.466	.000	.437	0	5	.0019	.24	250						
3	4	.4291	−.0033	.466	.021	.435	0	0	.0020	.21	236						
4	2	.4291	−.0033	.466	.000	.437	0	5	.0022	.18	229						
5	2	.4291	−.0033	.466	.021	.436	0	5	.0027				0.12	14,460	148	176	
6	4	.4295	−.0029	.466	.021	.437	0	0	.0020				.18	13,410	203		

1/2 Inch Dardelet

3	2	0.4390	−0.0010		0.133	0.627	0	29	0.0012	0.21	264	257					1.13
4	2	.4394	−.0006		.128	.626	0	14	.0010	.22	264						
5	3	.4394	−.0006		.112	.626	0	14	.0010	.20	243						
6	1	.4380	−.0020		.128	.627	0	14	.0018	.24	257						
1	3	.4394	−.0006		.133	.623	0	10	.0021				0.18	15,240	227	228	
2	1	.4390	−.0010		.117	.624	0	10	.0017				.20	13,350	230		

[1] Did not comply with requirements.

TABLE 8.—*Results of measurements and tests: bronze*—Continued

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Specimen no.	Bolt					Nut		Clearance between nut and bolt	Impact			Static				*Ratio* Impact work (col. 12) / Static work (col. 16)
	Bar no.	Actual minor diameter	Difference between actual and minimum minor diameter	Pitch diameter	Angle between axis of large thread and axis of bolt	Thickness	Obliquity of bearing face		Stretch, 5 diameters	Impact work		Stretch, 5 diameters	Maximum load	Static work		
										Each specimen	Average			Each specimen	Average	
		in.	in.	in.	deg	in.	deg min	in.	in.	ft-lb	ft-lb	in.	lb	ft-lb	ft-lb	
⅝ Inch American National Coarse																
2	5	0.4949	−0.0079	0.562	0.000	0.548	0 4	0.0045	0.11	250	302					1.37
3	3	.5028	.0000	.561	.000	.539	0 4	.0055	.16	346						
4	5	.4933	−.0095	.561	.000	.547	0 0	.0052	.14	250						
6	4	.5012	−.0016	.560	.000	.549	0 8	.0062	.22	362						
1	5	.5035	+.0007	.561	.000	.546	0 4	.0042				0.08	23,200	178	221	
5	4	.5028	.0000	.562	.022	.545	0 11	.0048				.16	19,170	264		
⅝ Inch American National Fine																
1	3	0.5555	+0.0057	0.587	0.000	0.544	0 8	0.0039	0.21	443	510					1.21
2	1	.5551	+.0053	.587	.000	.551	0 4	.0039	.31	555						
3	4	.5551	+.0053	.587	.000	.547	0 8	.0039	.31	555						
4	3	.5551	+.0053	.587	.038	.545	0 11	.0040	.29	485						
5	5	.5551	+.0053	.587	.000	.545	0 4	.0045				0.14	26,740	331	422	
6	4	.5567	+.0069	.586	.000	.545	0 27	.0055				.29	21,650	512		

⅝ Inch Dardelet

1	5	0.5504	+0.0004		0.147	0.752	0	0	0.0026	0.18	418	445					1.18
2	3	.5520	+.0020		.115	.751	0	19	.0013	.22	443						
3	3	.5507	+.0007		.093	.749	0	15	.0024	.20	434						
4	3	.5512	+.0012		.126	.752	0	11	.0009	.28	485						
5	4	.5504	+.0004		.071	.752	0	4	.0027				0.23	21,350	463	376	
6	5	.5507	+.0007		.104	.750	0	15	.0015				.12	25,940	290		

¾ Inch American National Coarse

1	1	0.6122	−0.0033	0.681	0.134	0.652	0	3	0.0041	0.28	697	573					1.22
2	5	.6110	−.0045	.681	.034	.654	0	7	.0054	.18	520						
3	1	.6043	−.0112	.682	.000	.657	0	3	.0038	.24	573						
4	3	.6094	−.0061	.681	.453	.650	0	0	.0043	.17	502						
5		.6106	−.0049	.682	.486	.650	0	3	.0040				0.26	27,680	598	469	
6	3	.6087	−.0068	.682	.000	.648	0	0	.0042				.12	32,680	340		

¾ Inch American National Fine

2	5	0.6701	+0.0045	0.707	0.039	0.654	0	3	0.0038	0.25	826	834					1.17
3		.6693	+.0037	.707	.078	.656	0	3	.0031	.25	856						
4		.6693	+.0037	.707	.000	.648	0	0	.0045	.30	866						
5	5	.6697	+.0041	.706	.179	.650	0	3	.0055	.23	788						
1	3	.6701	+.0045	.707	.000	.655	0	0	.0020				0.16	37,800	530	711	
6	4	.6701	+.0045	.707	.268	.648	0	3	.0037				.36	31,450	892		

¾ Inch Dardelet

1	4	0.6764	+0.0014		0.173	0.878	0	17	0.0015	0.32	954	874					1.27
2	4	.6750	.0000		.151	.872	0	7	.0017	.34	944						
3	3	.6756	+.0006		.168	.878	0	10	.0014	.20	711						
4	4	.6768	+.0018		.129	.880	0	3	.0007	.30	885						
5	5	.6764	+.0014		.123	.871	0	7	.0006				0.27	38,410	863	687	
6	5	.6768	+.0018		.173	.870	0	13	.0009				.15	38,590	511		

[1] Did not comply with requirements.

TABLE 9.—*Results of measurements and tests: brass*

1	2	3	4	5	6	7	8		9	10	11	12	13	14	15	16	17
⅜ Inch American National Coarse																	
Specimen no.	Bolt					Nut			Clearance between nut and bolt	Impact			Static				*Ratio* Impact work (col. 12) / Static work (col. 16)
	Bar no.	Actual minor diameter	Difference between actual and minimum minor diameter	Pitch diameter	Angle between axis of large thread and axis of bolt	Thickness	Obliquity of bearing face			Stretch, 5 diameters	Impact work: Each specimen	Impact work: Average	Stretch, 5 diameters	Maximum load	Static work: Each specimen	Static work: Average	
		in.	in.	in.	deg	in.	deg	min	in.	in.	ft-lb	ft-lb	in.	lb	ft-lb	ft-lb	
1	2	0.2862	−0.0043	0.332	0.094	0.322	0	0	0.0050	0.07	32	33					2.21
2	4	.2862	−.0043	.332	.078	.327	0	0	.0052	.06	32						
3	2	.2854	−.0051	.332	.083	.324	0	7	.0061	.08	37						
4	2	.2854	−.0051	.331	.094	.329	0	0	.0056	.07	32						
5	2	.2850	−.0055	.332	.104	.322	0	7	.0049				0.05	4,300	18.2	14.9	
6	2	.2862	−.0043	.332	.047	.321	0	7	.0053				.03	4,140	11.6		
⅜ Inch American National Fine																	
1	2	0.3197	+0.0013	0.347	0.089	0.328	0	26	0.0023	0.13	60	60					2.21
2	2	.3212	+.0028	.348	.099	.330	0	13	.0021	.12	60						
3	4	.3205	+.0021	.347	.094	.329	0	13	.0013	.11	60						
4	2	.3216	+.0032	.347	.063	.328	0	7	.0030	.11	60						
5	2	.3216	+.0032	.347	.089	.330	0	33	.0010				0.08	5,080	35.8	27.1	
6	4	.3181	−.0003	.346	.073	.328	0	26	.0027				.04	5,480	18.4		

⅜ Inch Dardelet

1	2	0.3267	+0.0017		0.177	0.499	0	46	0.0008	0.15	64	66					1.68
2	2	.3263	+.0013		.156	.499	0	20	.0009	.14	64						
3	2	.3263	+.0013		.250	.503	0	33	.0005	.15	70						
4	2	.3260	+.0010		.198	.498	0	20	.0009	.14	64						
5	2	.3263	+.0013		.172	.498	0	20	.0010				0.09	5,150	41.7	39.2	
6	2	.3267	+.0017		.162	.502	0	33	.0010				.08	5,240	36.8		

½ Inch American National Coarse

2	1	0.3957	−0.0006	0.450	0.112	0.438	0	5	0.0015	0.08	60	66					2.62
3	4	.3917	−.0046	.449	.059	.438	0	5	.0039	.09	64						
4	1	.3929	−.0034	.450	.053	.435	0	0	.0034	.09	70						
5	1	.3945	−.0018	.450	.064	.435	0	5	.0034	.12	70						
6	1	.3941	−.0022	.450	.053	.438	0	5	.0032				0.02	8,170	20.9	25.2	
1	1	.3937	−.0026	.450	.064	.441	0	5	.0019				.04	8,190	29.4		

½ Inch American National Fine

1	4	0.4303	−0.0021	0.466	0.075	0.430	0	5	0.0021	0.15	123	117					2.02
2	1	.4307	−.0017	.466	.059	.436	0	5	.0025	.16	128						
3	1	.4303	−.0021	.466	.080	.437	0	0	.0018	.16	123						
4	1	.4291	−.0033	.466	.096	.438	0	10	.0027	.13	95						
5	1	.4295	−.0029	[1] .469	.075	.435	0	10	.0016				0.07	9,180	59.8	58	
6	1	.4299	−.0025	.466	.053	.437	0	5	.0028				.07	9,160	56.1		

½ Inch Dardelet

3	1	0.4386	−0.0014		0.107	0.628	0	19	0.0028	0.16	134	136					1.84
4	1	.4390	−.0010		.101	.625	0	10	.0014	.18	140						
5	4	.4386	−.0014		.123	.625	0	19	.0014	.15	128						
6	1	.4394	−.0006		.320	.625	0	24	.0008	.19	140						
1	1	.4386	−.0014		.181	.628	0	19	.0026				0.09	9,570	77.5	74	
2	4	.4390	−.0010		.133	.627	0	19	.0029				.07	10,190	70.5		

[1] Did not comply with requirements.

TABLE 9.—*Results of measurements and tests: brass*—Continued

1	2	3	4	5	6	7	8		9	10	11	12	13	14	15	16	17
⅝ Inch American National Coarse																	
Specimen no.	Bolt					Nut			Clearance between nut and bolt	Impact			Static				Ratio Impact work (col. 12) / Static work (col. 16)
	Bar no.	Actual minor diameter	Difference between actual and minimum minor diameter	Pitch diameter	Angle between axis of large thread and axis of bolt	Thickness	Obliquity of bearing face			Stretch, 5 diameters	Impact work		Stretch, 5 diameters	Maximum load	Static work		
											Each specimen	Average			Each specimen	Average	
		in.	in.	in.	deg	in.	deg	min	in.	in.	ft-lb	ft-lb	in.	lb	ft-lb	ft-lb	
1	3	0.5051	+0.0023	[1] 0.568	0.022	0.546	0	0	0.0019	0.11	117	124					2.80
2	3	.5039	+.0011	[1] .567	.027	.546	0	0	.0021	.11	123						
3	3	.5047	+.0019	[1] .567	.016	.543	0	4	.0021	.13	134						
4	3	.5028	.0000	.565	.049	.541	0	4	.0046	.11	123						
5	3	.5024	−.0004	.565	.011	.548	0	0	.0034				0.04	12,820	45.6	44.2	
6	3	.5043	+.0015	[1] .567	.022	.544	0	4	.0018				.03	13,100	42.8		
⅝ Inch American National Fine																	
1	3	0.5476	−0.0022	0.587	0.033	0.546	0	4	0.0049	0.16	209	202					3.07
2	3	.5472	−.0026	.586	.000	.548	0	8	.0035	.16	209						
3	3	.5468	−.0030	.587	.000	.546	0	0	.0049	.15	196						
4	3	.5468	−.0030	.587	.044	.546	0	4	.0038	.15	196						
5	4	.5472	−.0026	.587	.027	.546	0	4	.0037				0.04	16,230	61.4	65.9	
6	3	.5468	−.0030	.587	.022	.547	0	4	.0048				.05	14,610	70.4		

⅝ Inch Dardelet

1	4	0.5512	+0.0012		0.131	0.753	0	11	0.0011	0.18	177	216					2.18
2	3	.5507	+.0007		.098	.750	0	4	.0013	.19	236						
3	3	.5507	+.0007		.164	.754	0	11	.0014	.20	236						
4	3	.5504	+.0004		.115	.753	0	8	.0022	.18	216						
5	3	.5516	+.0016		.109	.750	0	8	.0007				0.06	14,490	86.4	99.2	
6	4	.5512	+.0012			.755							.07	16,450	112		

¾ Inch American National Coarse

1	5	0.6079	−0.0076	0.681	0.022	0.651	0	0	0.0064	0.11	209	212					3.53
2	5	.6130	−.0025	[1].686	.000	.657	0	0	.0017	.13	196						
3	5	.6154	−.0001	.682	.084	.652	0	7	.0053	.13	229						
4	5	.6157	+.0002	.684	.000	.649	0	3	.0020	.14	216						
5	5	.6094	−.0061	.682	.000	.645	0	3	.0046				0.03	18,720	53.8	60.1	
6	5	.6083	−.0072	.682	.134	.655	0	3	.0054				.03	18,900	66.4		

¾ Inch American National Fine

1	5	0.6571		[1]0.703	0.073	0.657	0	3	0.0025	0.17	257	275					2.50
2	5	.6602	−0.0054	.707	.028	.651	0	0	.0052	.15	264						
3	5	.6590	−.0066	.707	.017	.655	0	0	.0059	.15	286						
4	5	.6622	−.0034	.707	.073	.653	0	0	.0055	.17	293						
5	5	.6610	−.0046	.707	.000	.655	0	7	.0061				0.06	21,650	118	110	
6	5	.6606	−.0050	.708	.000	.651	0	3	.0047				.05	21,540	101		

¾ Inch Dardelet

1	5	0.6728	−0.0022		0.140	0.875	0	13	0.0032	0.17	286	326					2.59
2	5	.6724	−.0026		.151	.873	0	13	.0023	.25	354						
3	2	.6740	−.0010		.140	.875	0	20	.0013	.22	370						
4	5	.6728	−.0022		.179	.878	0	20	.0027	.17	293						
5	5	.6724	−.0026		.157	.874	0	27	.0054				0.06	21,770	118	126	
6	5	.6740	−.0010		.112	.876	0	37	.0032				.06	22,130	134		

[1] Did not comply with requirements.

The thickness of each nut was measured using a micrometer caliper. All of the American National nuts, both coarse and fine, complied with the requirements as to thickness for semifinished regular nuts. No tolerances were given for the Dardelet nuts.

The obliquity of the bearing face of each nut was measured, using the fixture shown in figure 10. The nuts were screwed on a slightly tapered mandrel, a, having a corresponding thread. The mandrel was placed on the centers, b—b, and the dial micrometer, c, was clamped so that the micrometer contact rested on the bearing face of the nut. The mandrel was then rotated by hand. The obliquity in degrees was computed from the difference between the maximum and minimum micrometer readings. If the difference was zero, the bearing face was perpendicular to the axis of the threads. All the nuts complied with the requirements for both semifinished and finished regular nuts.

The diametrical clearance between the bolt and nut was measured using the fixture shown in figure 11. The bolt was held firmly in the fixture by the clamping plate, a, and thumbscrew, b. The spindle of the dial micrometer rested on the nut directly over the axis of the bolt. A small upward force was applied to the nut by the fingers, while the nut was rotated back and forth through a small angle, the minimum reading being recorded. This operation was repeated with a small downward force applied to the nut. The difference between the two readings was taken as the diametrical clearance between the bolt and nut. In measuring the clearance between a bolt and nut having Dardelet threads the same procedure was followed except that a small axial force, toward the head of the bolt, was applied to the nut in addition to the upward and downward forces. This axial force kept the nut in the position of maximum clearance.

Insofar as could be determined from these measurements of clearance all of the American National specimens, both coarse and fine, complied with the requirements for pitch-diameter tolerance for class 3, medium fit. No tolerances were given for the Dardelet specimens.

V. METHOD OF TESTING

1. GENERAL

As the results of impact tests nearly always show greater variations than the results of static tests, four of the six similar specimens of each material, thread, and size were tested under impact tensile loading and the other two under static tensile loading. The distance between the shoulder on the bolt and the bearing face of the nut was five times the nominal diameter of the bolt. In this distance, the thread was exposed one diameter from the face of the nut, as shown in figure 9.

2. IMPACT TESTS

The impact tests were made in the Charpy machine shown in figures 12, 13, and 14, located at Watertown Arsenal, Mass. The capacity was 2,170 ft-lb (300 kg-m).

As shown in figure 14, the bolt, a, was screwed into the pendulum, b, the spacer, c, and the collar, d, were placed on the bolt, then the nut, e, was screwed on hand-tight. If the nuts on the Dardelet

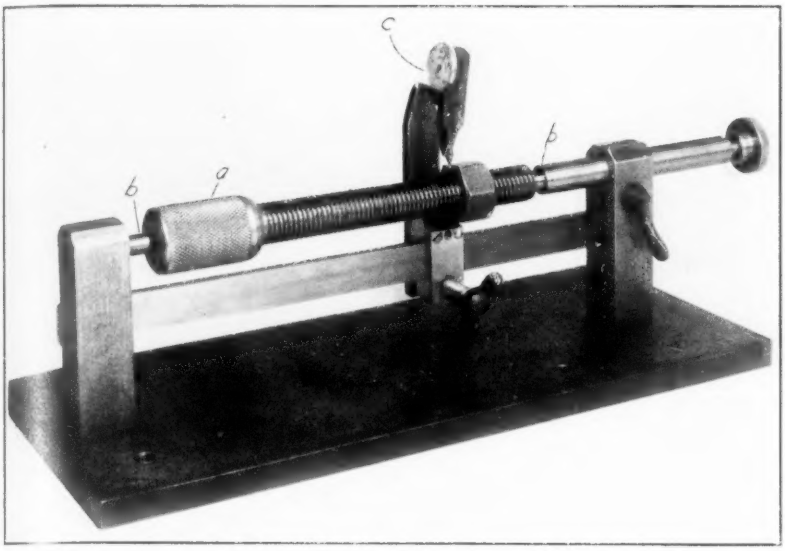

FIGURE 10.—*Fixture used for measuring the obliquity of the face of the nut.*

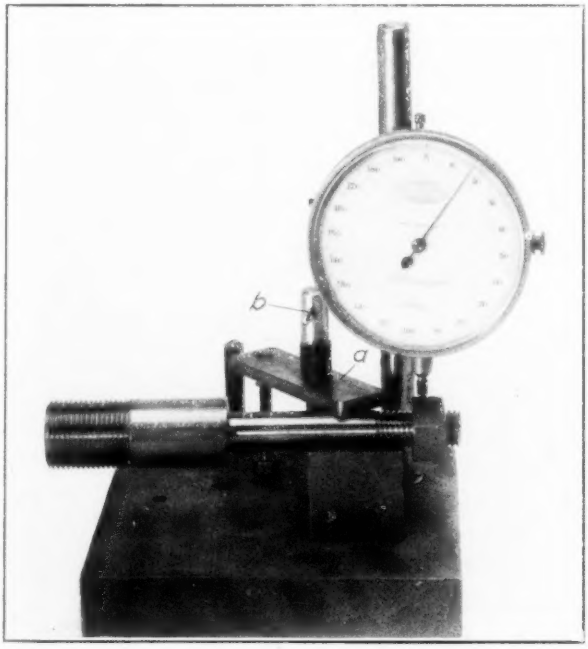

FIGURE 11.—*Fixture used for measuring the diametrical clearance between the threads of the bolt and the threads of the nut.*

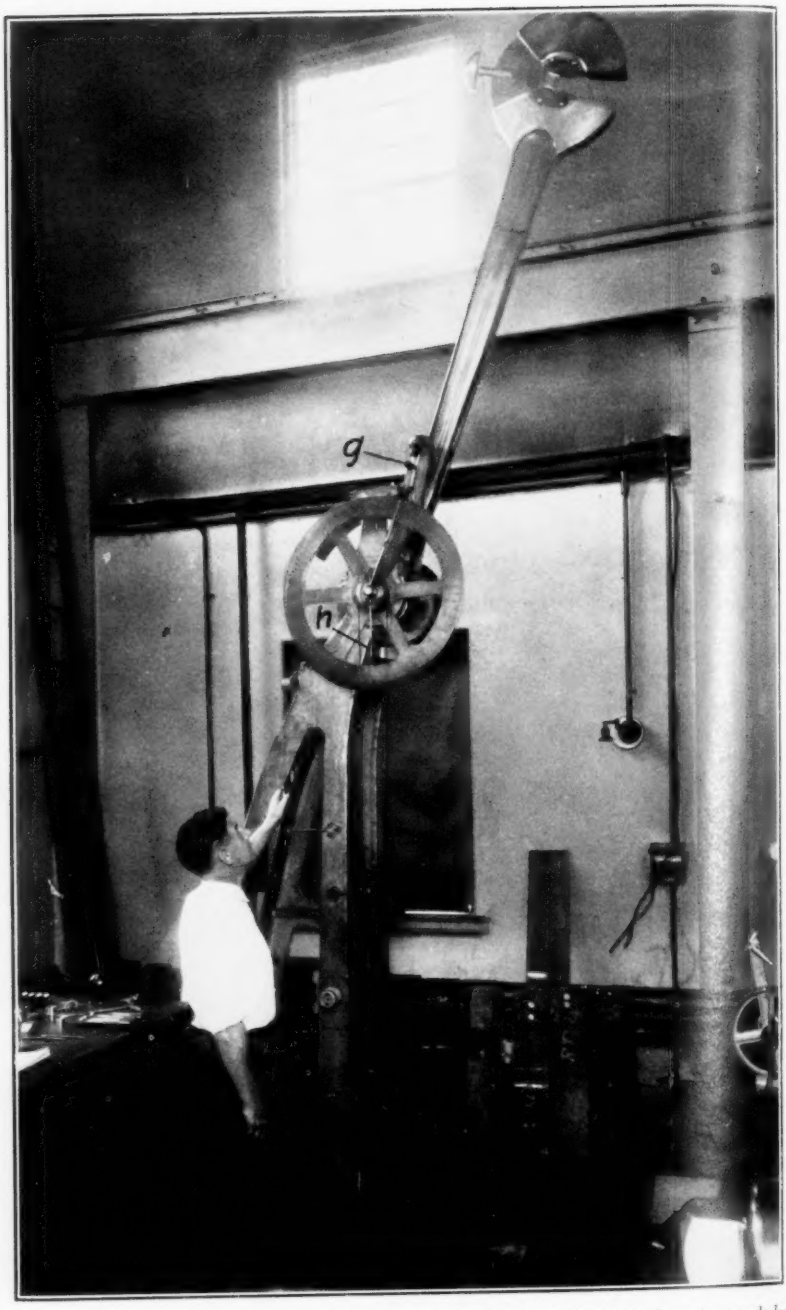

FIGURE 12.—*Charpy impact machine, capacity 2,170 ft-lb (300 kg-m), pendulum raised.*

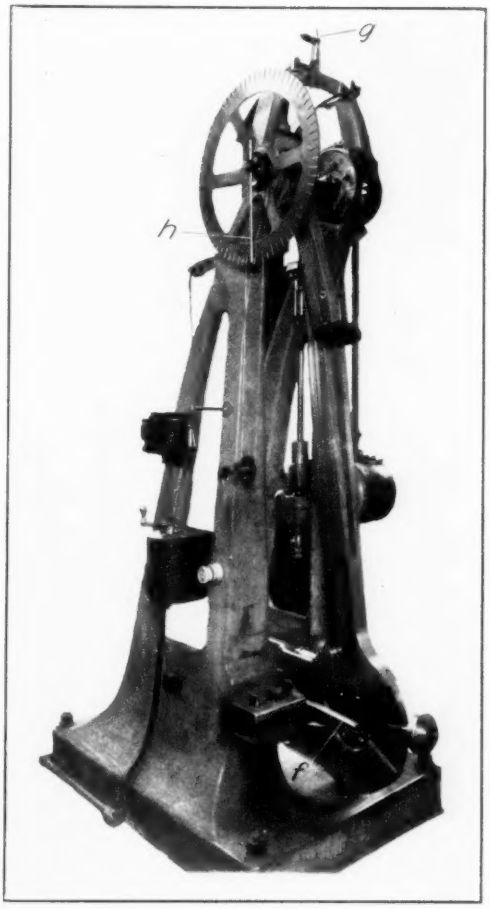

FIGURE 13.—*Charpy impact machine, collar near the stops.*

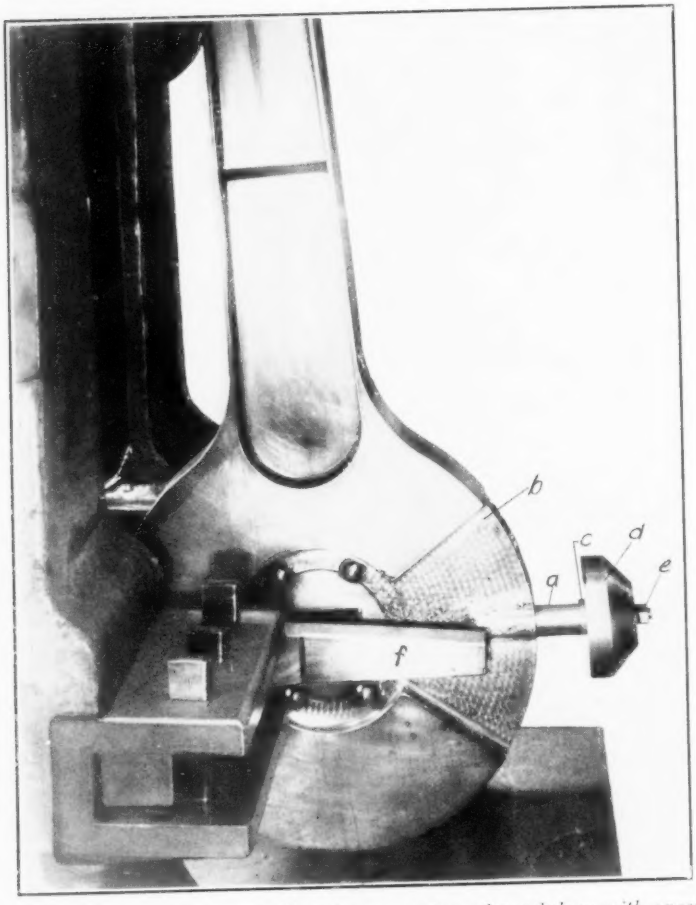

FIGURE 14.—*Charpy impact machine, lower portion of pendulum with specimen in position.*

specimens had been tightened as in actual use, the thrust faces of the threads would have been in contact. The work required to bring the thrust faces into contact was estimated from the load-stretch diagrams for the ½-in. bolts. For the chromium-nickel steel and the Monel-metal specimens this work was less than 1 percent of the area of the diagram and, therefore, of the static work. For the bronze and the brass specimens, the work was less than 1½ percent and for the cold-rolled steel specimens less than 2¾ percent. The values of the impact work should be decreased by about these percentages when estimating the impact work of Dardelet bolts which are under service tensile stress. The work required for American National bolts, both coarse and fine, to subject them to service tensile stress is much less than the work required for Dardelet bolts. Approximate computations based on the results of the static tests on ½ in. bolts showed that this work was less than one-half of 1 percent of the static work. For some of the steel specimens it was less than 1 percent of the static work.

A different spacer, c, was used for the specimens of each nominal diameter. The spacer was machined so that the sum of its thickness and the thickness of the collar was equal within a tolerance of 0.02 in. to five times the nominal diameter of the bolt.

If there was a measurable angle between the axis of the large thread and the axis of the bolt, the bolt was rotated in the pendulum until the axis lay in a vertical plane. The collar, therefore, struck both stops, f, simultaneously. A prick punch was used to mark the position of the nut on the bolt.

The pendulum was raised until it engaged the latch release, g, figures 12 and 13, where it was held at an angle of 70° above the horizontal. The pointer, h, was set at zero and the latch tripped, releasing the pendulum and allowing it to swing downward. The collar struck the stops and the specimen ruptured. Due to its residual energy, the pendulum continued past the vertical position. The pointer registered the maximum angle of swing after rupturing the bolt. This angle was read to the nearest ½° and the residual energy computed. The impact work required to rupture the specimen was obtained by subtracting the residual energy of the pendulum from its initial energy.

Some of the energy was expended in stretching the portion of the bolt which had not been turned down. The length of this portion differed for each size of specimen because the overall lengths of the bolts were the same and the length of the reduced portion was five times the nominal diameter or size. Approximate calculations based upon the materials having the highest ratio of ultimate strength to Young's modulus of the material and the greatest static load sustained by any specimen showed that this work could in no case exceed 2 percent of the total impact work for any of the ¾ in. specimens. This percentage would decrease as the nominal diameter of the specimen decreased.

With the nut in the position indicated by the punch mark, the length of the ruptured specimens from the shoulder on the bolt to the bearing face of the nut was measured. The stretch was obtained by subtracting the initial distance (five times the nominal diameter of the bolt) from this distance.

3. STATIC TESTS

The static specimens were tested in a screw-power, beam and poise testing machine having a capacity of 50,000 lb. The extensometer shown in figure 15 was attached to the specimen and simultaneous readings of the load and dials taken until the stretch increased much more rapidly than the load. The extensometer was then removed to prevent injuring the dial micrometers. Two micrometers, a–a, mounted on a collar were attached to the bolt by three screws, one-half of the diameter of the bolt above the shoulder. The clamp, b, was fastened by horizontal knife edges to the nut, one-half of the nominal diameter of the bolt below the bearing face of the nut. The gage length was therefore six times the nominal diameter of the bolt. The spindles of the micrometers rested on the horizontal upper surface of the clamp. The dial micrometers were calibrated by using a bench screw micrometer. It is believed that the error of the extensometer did not exceed 0.001 in.

The stretch obtained from the extensometer readings was taken as the stretch between the shoulder on the bolt and the face of the nut because the deformations at the shoulder of the bolt and of the nut were so small as to be negligible.

Before the extensometer was removed, the points of a modified divider shown in figure 16 were applied to the specimen and the stretch for several loads was read simultaneously on both extensometer and divider. The points of the divider were placed in prick punch marks on the specimen. These marks were in the horizontal planes at which the extensometer was attached. The stretch of the specimen was indicated on the scale, a, by the lever, which had a ratio of 10:1. This divider was calibrated using a bench screw micrometer. It is believed that the error of the readings of the divider did not exceed 0.005 in. Readings of the divider and corresponding loads were taken as the stretch increased until the specimen ruptured.

VI. RESULTS

The results of the measurements and tests of the specimens are given in tables 5 to 9, inclusive.

From the load-stretch readings, diagrams were plotted for each static specimen. Diagrams for one of the two duplicate specimens of all materials having nominal diameters of ½ in. are shown in figure 17. These are typical of those for the other specimens. For all the specimens having American National fine and Dardelet threads, the stretch was greater than for those having American National coarse threads. For low loads the curves of the American National threads, coarse and fine, show that the stretch is approximately proportional to the load. For loads up to a few thousand pounds the stretch of the Dardelet specimens was much greater than that for the specimens having American National threads, due to axial movement of the nut on the bolt until the thrust faces came into contact. For higher loads, after the thrust faces came into contact, the stretch increased at about the same rate as for the specimens having American National threads. The static work was computed from the area under the load-stretch curve.

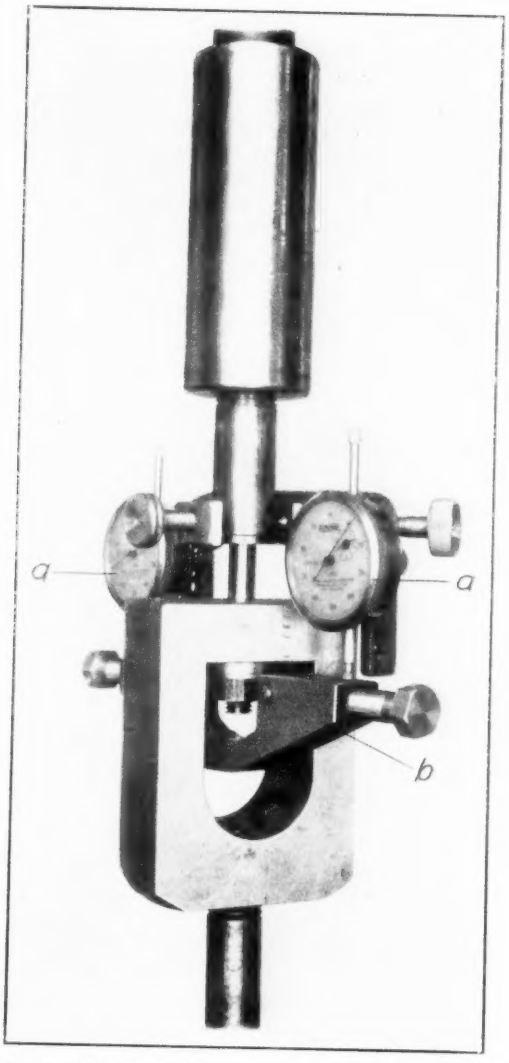

FIGURE 15.—*Extensometer used for measuring elastic stretch in static tensile specimens.*

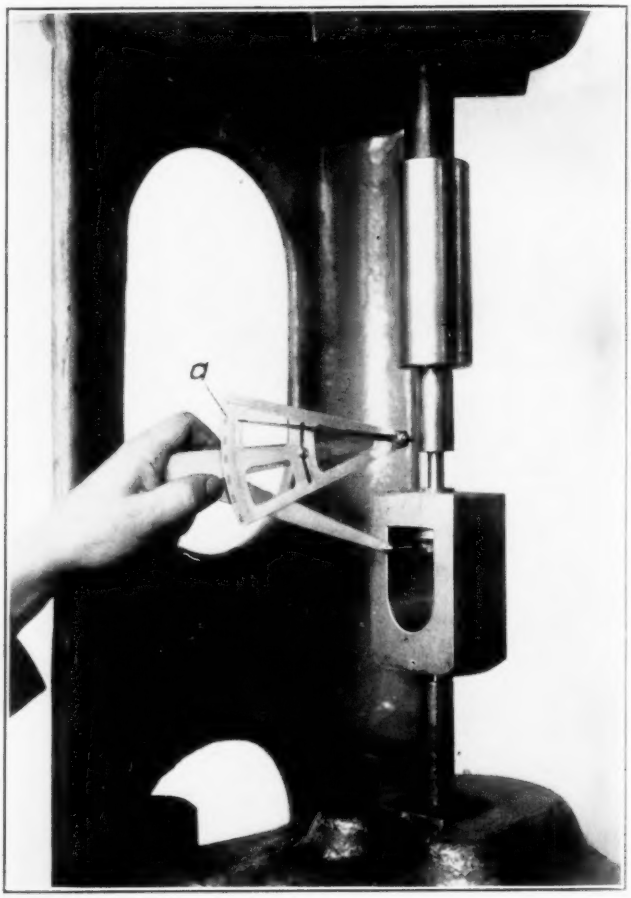

FIGURE 16.—*Dividers used for measuring the stretch to rupture in static tensile specimens.*

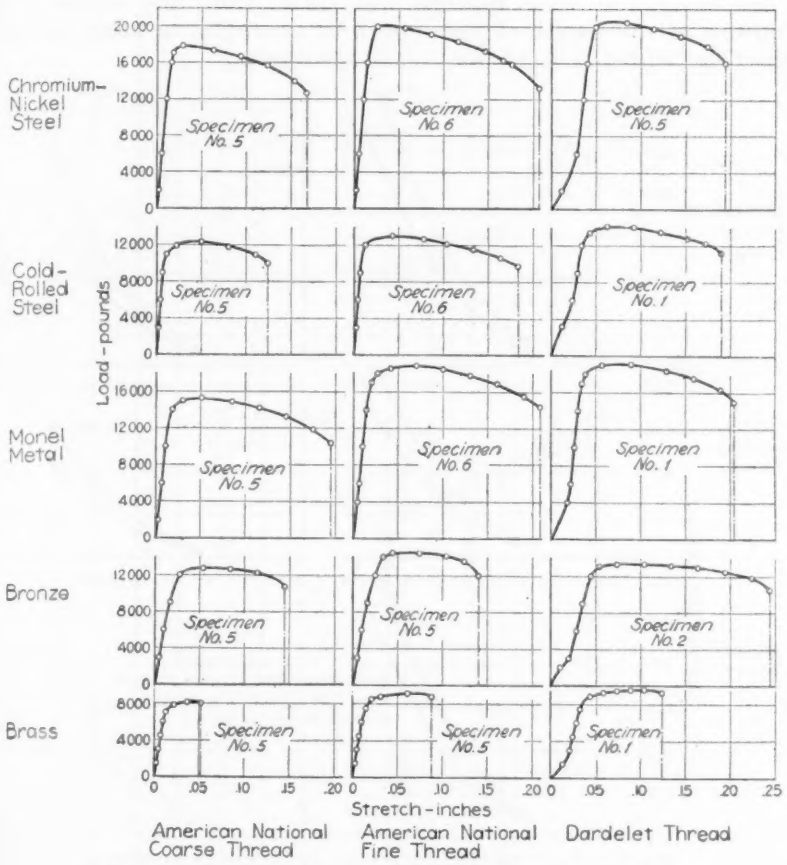

FIGURE 17.—*Static load-stretch diagrams for ½-inch bolts.*

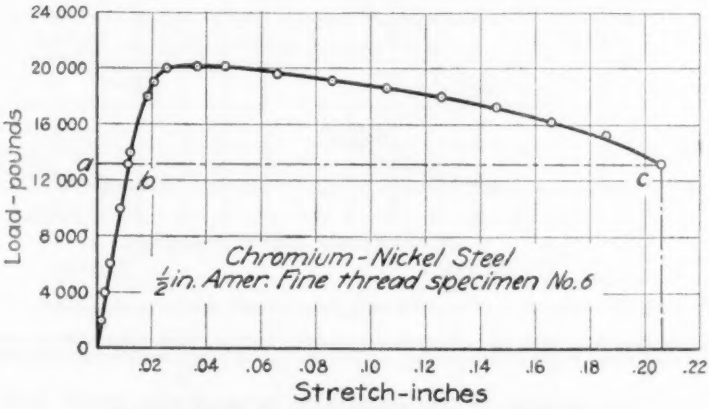

FIGURE 18.—*Typical static load-stretch diagram.*

The elongation after rupture was taken as the length of the line, b–c, to allow for the elastic shortening a–b, of the broken specimen.

The elongation after rupture was taken from the load-stretch diagrams as the length of the line, b–c, figure 18, to allow for the elastic shortening, a–b, of the broken specimen.

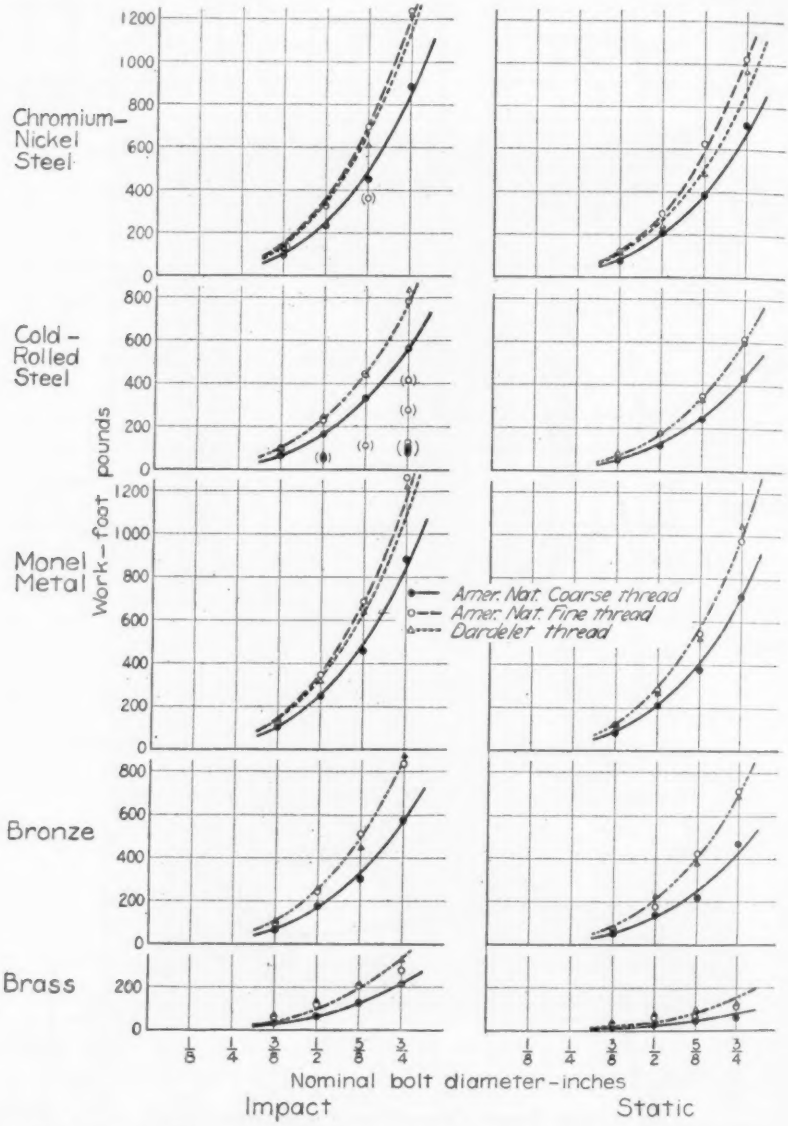

FIGURE 19.—*Relation of impact and static tensile works to bolt diameter.*

The values in parentheses were not included in the average. Impact.—Each point is the average work for 4 specimens, except for the steel specimens. Static.—Each point is the average work for 2 specimens.

The average values of the impact work and the static work are shown graphically in figure 19, and the average values of the maximum static loads in figure 20.

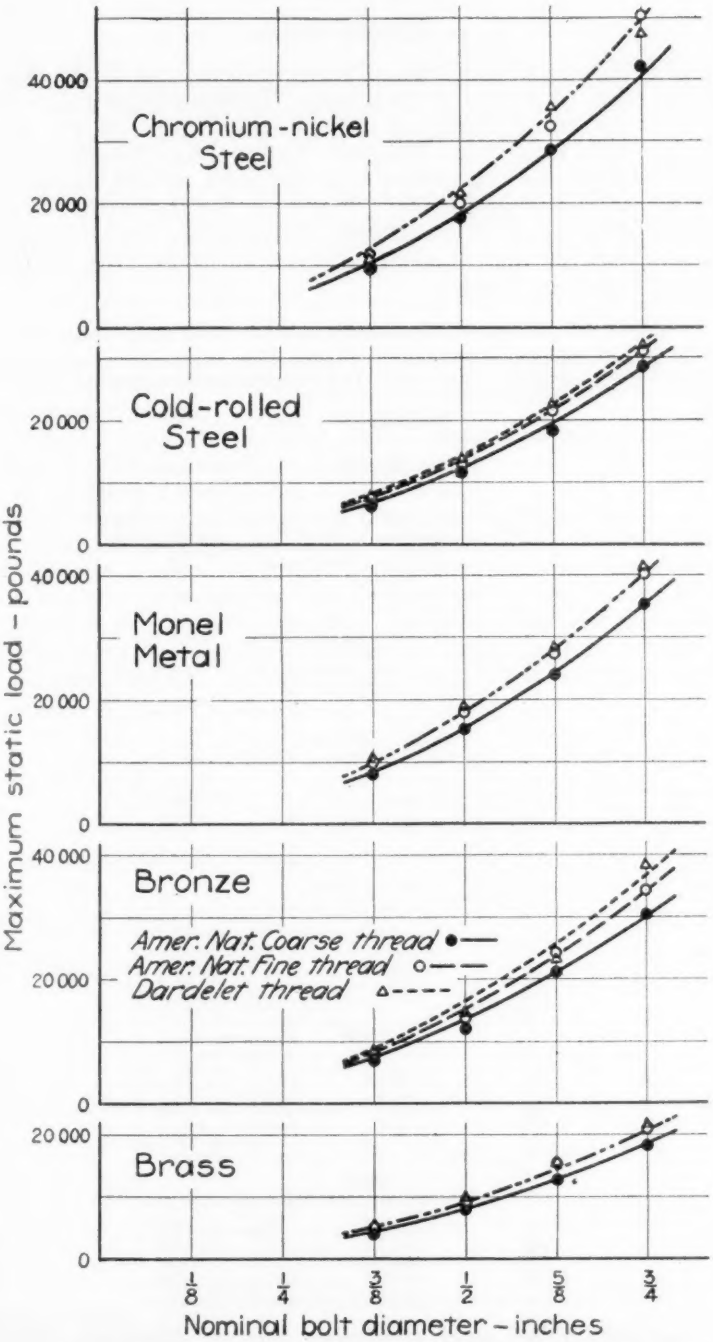

FIGURE 20.—*Relation of maximum static loads to bolt diameters.*

Each point is the average maximum load for 2 specimens.

VII. DISCUSSION

1. IMPACT TESTS

Of the 240 impact specimens all but two fractured in the threads. These two failed by stripping the threads. They were chromium-nickel steel, ⅜-in. American National fine thread, specimen no. 2, and chromium-nickel steel, ⅝-in. American National fine thread, specimen no. 1. The work for the former was the same as the lowest value for the other similar specimens. The work for the latter, however, was only 53 percent of the lowest value for the other similar specimens. No explanation of this unusual behavior of these two chromium-nickel specimens was found.

TABLE 10.—*Differences in impact works for similar specimens*

[The average impact works are given in tables 5 to 9, inclusive. The differences not in parentheses are only for the specimens included in the average. The differences in parentheses are for all the specimens including those having stripped threads and brittle failures.]

Nominal diameter	Maximum differences from the average		
	American National coarse thread	American National fine thread	Dardelet thread
Inch	*Percent*	*Percent*	*Percent*
Chromium-Nickel Steel			
⅜	+6.4	−3.0(−3.0)	+5.3
½	+2.2	−2.8	±2.4
⅝	+7.5	+3.6(−47.9)	−8.3
¾	−3.1	+4.6	−6.9
Cold-Rolled Steel			
⅜	−5.9	+3.3	±5.3
½	0.0(−69.5)	±4.4	±2.9
⅝	±4.5	−2.5(−75.0)	+5.7
¾	0.0(−86.0)	0.0(−83.3)	+2.6
Monel Metal			
⅜	0.0	±2.3	±2.3
½	−3.7	+2.9	−7.6
⅝	+2.9	−8.2	±1.4
¾	−2.7	−2.5	+1.6
Bronze			
⅜	−19.4	+8.7	−3.1
½	−4.0	−5.0	−5.4
⅝	+19.9	−13.1	+9.0
¾	+21.6	−5.5	−18.6
Brass			
⅜	+12.1	0.0	+6.1
½	−9.1	−18.8	−5.9
⅝	+8.1	+3.5	−18.0
¾	+8.0	±6.5	+13.5

For the cold-rolled steel specimens having American National threads, coarse and fine, the impact works were very much more erratic than those for the specimens of other materials. For ten of the cold-rolled specimens the fracture was like that of a brittle

material and the impact work was much lower than that of the similar specimens. Typical specimens are shown in figure 21. For each pair the one on the left had a brittle failure and the one on the right a ductile failure. It is evident that the reduction of area was much greater for the specimens on the right. Metallurgical examination of these cold-rolled steel specimens gave no conclusive reason for the differences in behavior. A careful study of the threads on representative specimens, particularly of the minor diameter and the profile and surface of the thread at the root, suggested no explanation. However, the fact that none of the Dardelet specimens showed brittle failures suggests that there might have been a notch effect under impact loading for the specimens having American National threads.

The maximum percentage differences from the average impact works for each group of similar specimens are given in table 10. As noted above, very great differences were found in the brittle failures of the cold-rolled steel, and in one of the stripped-thread failures of the chromium-nickel steel.

Excluding these failures, all of which occurred in specimens having American National threads, the remaining differences were about the same for all three types of thread.

The bronze and brass specimens were the least uniform.

The ratios of the average impact works are given in table 11. For all of the materials, the impact work for bolts having American National coarse threads was less than the impact work for bolts of the same size and material having American National fine threads. The ratios for all of the bolts except the brass bolts were about the same. The ratios for the brass bolts were somewhat lower than for bolts of the other materials.

TABLE 11.—*Ratios of impact works*

Specimen		Ratios of average impact work		
Nominal diameter	Material	American National coarse: American National fine	American National fine: Dardelet	Dardelet: American National coarse
Inch	Chromium-nickel steel	[1] 0.71	[1] 0.99	1.42
	Cold-rolled steel	.74	.97	1.40
¾	Monel metal	.76	1.00	1.31
	Bronze	.67	.94	1.58
	Brass	.55	.91	1.97
	Chromium-nickel steel	.71	.96	1.46
	Cold-rolled steel	[2] .73	.93	[3] 1.48
½	Monel metal	.71	1.09	1.29
	Bronze	.73	.94	1.45
	Brass	.56	.86	2.06
	Chromium-nickel steel	[1] .65	[1] 1.15	1.34
	Cold-rolled steel	[2] .74	[2] 1.02	1.31
⅝	Monel metal	.66	1.07	1.41
	Bronze	.59	1.15	1.47
	Brass	.61	.93	1.74
	Chromium-nickel steel	.71	1.02	1.38
	Cold-rolled steel	[2] .73	[2] .92	[3] 1.48
¾	Monel metal	.70	1.03	1.39
	Bronze	.69	.95	1.53
	Brass	.77	.84	1.54
	Chromium-nickel steel	[1] .70	[1] 1.03	1.40
	Cold-rolled steel	[2] .74	[2] .96	[3] 1.42
Average for all diameters	Monel metal	.71	1.05	1.35
	Bronze	.67	1.00	1.51
	Brass	.62	.88	1.83

[1] Stripped threads not included in the average (see table 5).
[2] Brittle failures not included in average (see table 6).

The impact work for bolts having American National fine threads was sometimes greater and sometimes less than for bolts having Dardelet threads.

For all of the materials the impact work for bolts having Dardelet threads was greater than the impact work for bolts of the same size and material having American National coarse threads. The ratios for the brass bolts were considerably greater than the ratios for the bolts of the other materials.

2. STATIC TESTS

Of the 120 static specimens, all fractured in the bolt threads. There were no brittle failures and in no case did the threads strip.

The maximum percentage differences from the average static works for similar specimens are given in table 12. For each of the threads these differences were small for the chromium-nickel steel, cold-rolled steel, and monel metal specimens. The differences for the bronze and the brass bolts were considerably greater.

TABLE 12.—*Differences in static works for similar specimens*

Nominal diameter	Differences from the average [1]		
	American National		Dardelet thread
	Coarse thread	Fine thread	
Inch	*Percent*	*Percent*	*Percent*
Chromium-Nickel Steel			
3/8	1.3	1.7	0.9
1/2	5.4	1.7	9.1
5/8	.3	.2	2.9
3/4	1.3	10.1	3.7
Cold-Rolled Steel			
3/8	0.2	0.3	1.2
1/2	2.5	1.1	3.3
5/8	5.3	2.0	3.9
3/4	1.4	.8	3.2
Monel Metal			
3/8	1.2	3.3	0.8
1/2	4.3	4.3	2.6
5/8	2.7	3.0	2.1
3/4	.3	.1	1.4
Bronze			
3/8	15.7	0.5	5.4
1/2	2.2	15.9	.9
5/8	19.4	21.6	23.1
3/4	27.5	25.9	25.6
Brass			
3/8	22.1	32.1	6.4
1/2	17.0	3.3	4.7
5/8	3.2	6.8	12.9
3/4	10.5	8.2	6.3

[1] As there were two similar static specimens the positive and the negative differences were the same numerically.

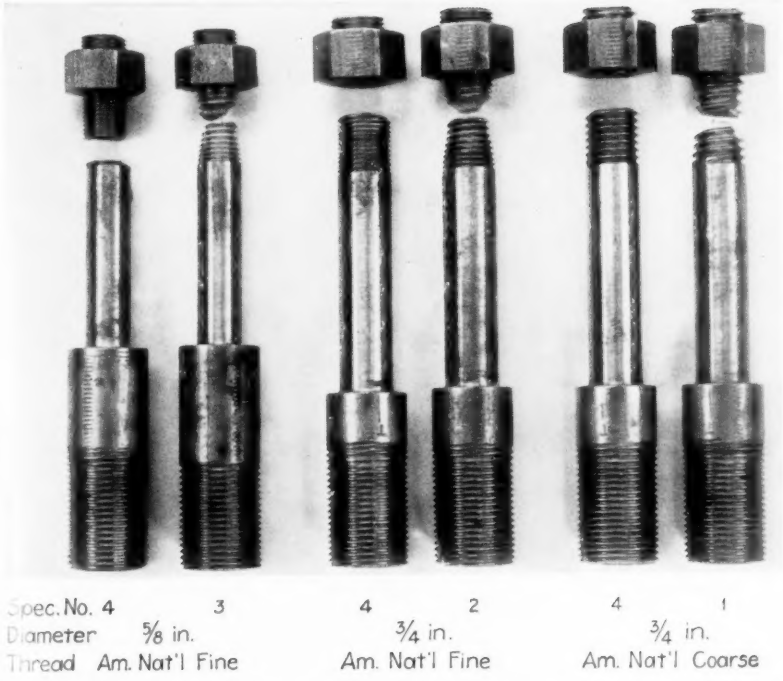

FIGURE 21.—*Cold-rolled steel specimens showing, for each pair, typical brittle failures on the left and typical ductile failures on the right.*

The ratios of the average static works and of the average maximum static tensile loads are given in table 13. For all of the materials the static work for bolts having American National coarse threads was less than the static work for bolts of the same size and material having American National fine threads. The ratios for all of the bolts except the brass bolts were about the same. The ratios for the brass bolts were somewhat lower than for bolts of the other materials. The static work for bolts having American National fine threads was sometimes greater and sometimes less than for bolts having Dardelet threads. For all of the materials the static work for bolts having Dardelet threads was greater than the static work for bolts of the same size and material having American National coarse threads. The ratios for the brass bolts were much greater than the ratios for the bolts of the other materials.

TABLE 13.—*Ratios of static works and maximum static loads*

Specimen		Ratios of average static work			Ratios of average maximum static loads		
Nominal diameter	Material	American National coarse: American National fine	American National fine: Dardelet	Dardelet: American National coarse	American National coarse: American National fine	American National fine: Dardelet	Dardelet: American National coarse
Inch							
⅜	Chromium-nickel steel	0.65	1.03	1.50	0.82	1.04	1.17
	Cold-rolled steel	.67	.93	1.60	.88	.98	1.16
	Monel metal	.68	.99	1.48	.82	1.00	1.22
	Bronze	.69	.94	1.54	.90	.94	1.18
	Brass	.55	.69	2.63	.80	1.02	1.23
½	Chromium-nickel steel	.68	1.25	1.18	.88	.94	1.20
	Cold-rolled steel	.66	.99	1.53	.91	.97	1.13
	Monel metal	.74	1.06	1.27	.85	.99	1.20
	Bronze	.76	.77	1.70	.86	.97	1.19
	Brass	.43	.78	2.94	.89	.93	1.21
⅝	Chromium-nickel steel	.61	1.29	1.26	.89	.91	1.24
	Cold-rolled steel	.70	1.05	1.35	.85	.99	1.19
	Monel metal	.69	1.05	1.38	.87	.98	1.16
	Bronze	.52	1.12	1.70	.88	1.02	1.12
	Brass	.67	.66	2.24	.84	1.00	1.19
¾	Chromium-nickel steel	.70	1.07	1.35	.83	1.06	1.13
	Cold-rolled steel	.70	1.03	1.39	.92	.99	1.10
	Monel metal	.73	.93	1.46	.88	.98	1.17
	Bronze	.66	1.03	1.46	.87	.90	1.28
	Brass	.55	.87	2.10	.87	.98	1.17
Average for all diameters	Chromium-nickel steel	.66	1.16	1.32	.86	.98	1.18
	Cold-rolled steel	.68	1.00	1.47	.89	.98	1.14
	Monel metal	.71	1.01	1.40	.86	.99	1.19
	Bronze	.66	.96	1.60	.88	.96	1.19
	Brass	.55	.75	2.48	.85	.98	1.20

For all of the materials, the maximum static load for bolts having American National coarse threads was less than the load for bolts of the same size and material having American National fine threads. The maximum static loads for bolts having American National fine threads were about the same as those for bolts having Dardelet threads. For all of the materials, the maximum static load for bolts having Dardelet threads was greater than the load for bolts of the same size and material having American National coarse threads.

The ratios for the maximum static loads for all of the materials were about the same.

The static efficiency of a bolt may be defined as the ratio of the maximum tensile load of the bolt to the maximum tensile load of the shank of the bolt. The latter was computed by multiplying the ultimate strength of the material by the cross-sectional area of the shank. The average bolt efficiencies and the ratios of the area at the root of the thread to the area of the shank are given in table 14. For all of the materials the bolt efficiency is greater than the ratios of the areas, showing that the tensile strength of the threaded portion of the bolts was greater than that of a cylindrical specimen having a diameter the same as the minor diameter of the bolt.

TABLE 14.—*Static bolt efficiencies*

Nominal diameter	Thread	Material					Root area, shank area
		Chrome-nickel steel	Cold-rolled steel	Monel metal	Bronze	Brass	
		Bolt efficiency [1]					
Inch		*Percent*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*
⅜	American National coarse	65.3	70.6	70.3	70.3	65.2	61.4
	American National fine	79.2	80.2	85.3	78.0	81.5	73.2
	Dardelet	76.4	82.2	85.7	83.2	80.2	75.1
½	American National coarse	69.1	73.5	74.3	67.3	71.1	64.0
	American National fine	78.1	80.5	87.9	77.9	79.7	75.6
	Dardelet	83.2	83.1	88.9	79.9	85.8	77.4
⅝	American National coarse	71.7	72.3	73.2	75.8	72.1	65.8
	American National fine	80.9	85.5	83.8	86.6	85.8	78.2
	Dardelet	88.5	86.1	85.1	84.6	86.0	77.4
¾	American National coarse	73.0	78.4	74.4	75.0	72.7	68.4
	American National fine	88.0	85.2	85.0	86.0	83.4	79.5
	Dardelet	82.8	86.3	87.0	95.7	84.8	81.0

[1] The ratio of the maximum tensile load of the bolt to the computed maximum tensile load of the shank of the bolt.

3. COMPARISON OF IMPACT AND STATIC PROPERTIES

Inspection of the broken specimens showed that most of the stretch occurred in the threaded portion one diameter in length between the bearing face of the nut and the shoulder of the bolt. Consequently, most of the work was absorbed by this portion. The work absorbed would be expected to be proportional to the product of this "effective length" by some area approximating the area at the root of the thread. For comparison it was decided to compute the "specific work" for both impact and static specimens as the ratio of the average values of the work given in tables 5 to 9 by the "effective root volume" taken as the length, 1 nominal diameter, times the cross-sectional area at the root of the thread computed from the nominal minor diameter. The specific work for the impact and the static specimens was then obtained by dividing the average values of the work by the effective root volume. The "effective stretch" was obtained by dividing the average stretch for similar specimens by the nominal diameter. The values for specific work and effective stretch are shown in figure 22.

The average values, for each material, of effective stretch and specific work for the bolts having American National fine threads are about the same as those for bolts having Dardelet threads. Except for the brass bolts the values for each material and type of thread

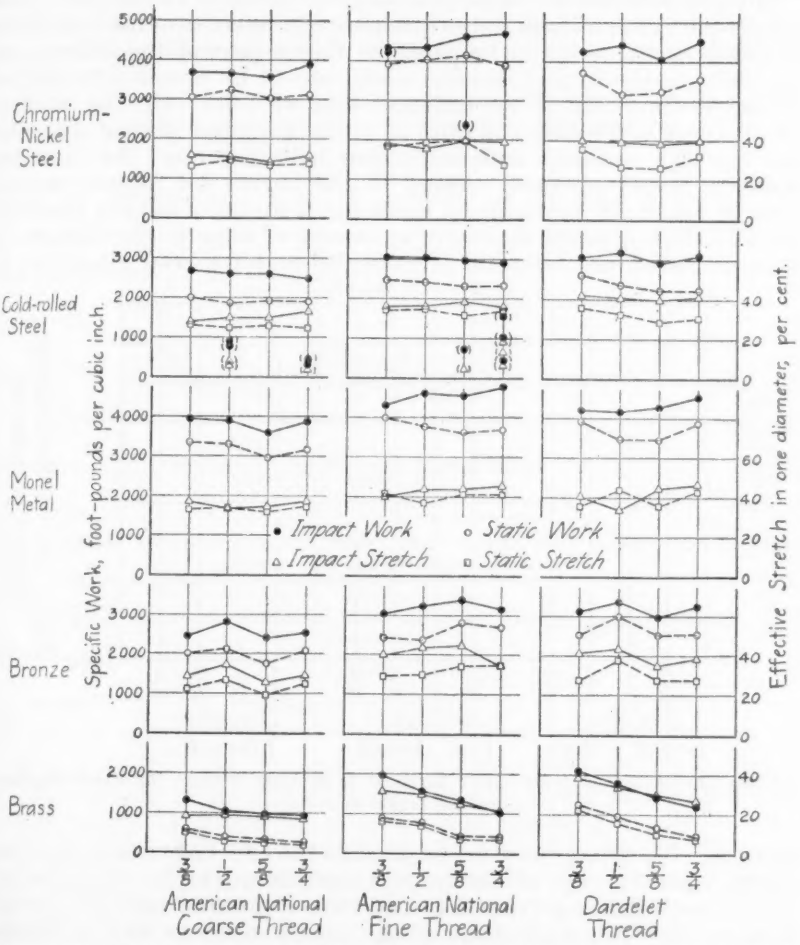

FIGURE 22.—*Relation between the size of the bolt and the specific work and effective stretch.*

The values in parentheses were not included in the average.

are about the same for the different bolt diameters. For the brass bolts the decrease in these values with an increase in the nominal diameter of the bolt probably is due to the fact that the hardness of the brass bars for the bolts decreased continuously toward the axis of the bar. Vickers numbers at points spaced 1/16 in. along two mutually perpendicular diameters on cross sections of the brass bars showed that the center was much softer than the cylindrical surface. The amount of the harder material was greater for the bolts having the larger diameters than for those having the smaller diameters. Some of the other materials also were somewhat softer at the center but the

differences were not nearly so great as for the brass. Six of the 8 bars of chromium-nickel steel were somewhat harder at the center, which may explain the small increase in impact specific work for these bolts with an increase in diameter.

The irregularities in the curves shown in figure 22 are probably due in large part to variations in the mechanical properties of the materials. However, there seems to be evidence that a part of the differences, especially in the impact specific work, are to be ascribed to lack of geometric similarity of the different sizes of bolts. On the average for all types of threads the ratio of minor to major thread diameter (see fig. 23) increases with increasing bolt diameter; the increase, however, is not uniform. Many of the curves for impact specific work in figure 22, particularly those for ⅝ in. bolts having Dardelet threads, show a rough similarity in change of slope to the changes in slope of the curves for ratio of minor to major thread diameters in

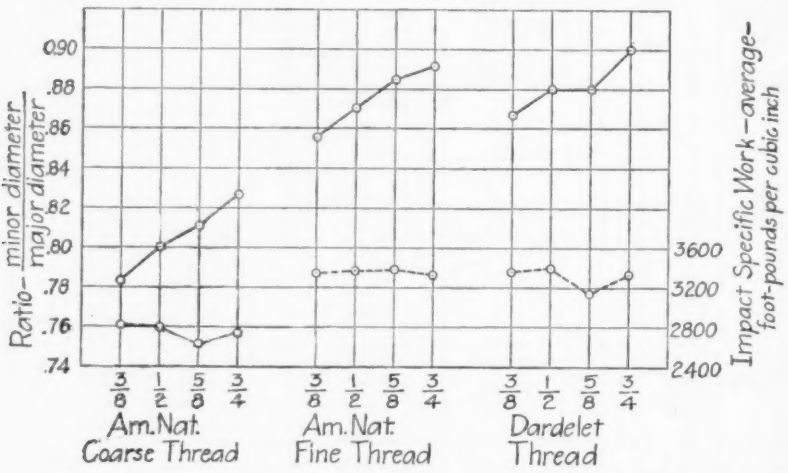

FIGURE 23.—*Relation of the major diameter to the ratio between the minor diameter and the major diameter.*

figure 23. To eliminate so far as possible the effect of variations in the material the average of the impact specific works for all materials and for each size and type of thread were computed and also plotted in figure 23. The comparison of the curves seems to show a definite relationship between impact specific work and ratio of minor to major thread diameter. Similar but less definite relationships are to be noticed in the static specific work, impact effective stretch, and static effective stretch. The variations in the individual values in all cases are too great to allow of a definite numerical evaluation of this effect of lack of similarity. For this reason it was decided to use the average of all sizes of bolts in making further comparisons.

The values in figure 22 were obtained from bolts having one diameter of thread exposed. If the thread had been exposed for a greater distance it is probable that the effective stretch and the specific work would have been greater.

The ratios of the average impact work to the average static work and the ratios of the average impact stretch to the average static stretch for all types of thread and all sizes of bolt are given in table 15.

For the brass bolts the ratio of the stretches was greater than the ratio of works indicating that on the average the tensile loads were less under impact than under static loading. For the bronze bolts the ratios were about the same indicating that the tensile loads were about the same. For the steel and the monel metal bolts, the ratios of the works are greater than the ratios of the stretches, indicating that probably the tensile loads were greater under impact than under static loading.

TABLE 15.—*Ratios of impact properties to static properties*

Material	Thread	Impact stretch / Static stretch	Impact work / Static work
Chromium-nickel steel	Coarse	1.117	1.190
	Fine	1.013	1.130
	Dardelet	1.312	1.268
Average		1.147	1.196
Cold-rolled steel	Coarse	1.207	1.348
	Fine	1.091	1.252
	Dardelet	1.282	1.305
Average		1.193	1.302
Monel metal	Coarse	1.086	1.210
	Fine	1.083	1.215
	Dardelet	1.183	1.170
Average		1.117	1.198
Bronze	Coarse	1.270	1.282
	Fine	1.276	1.250
	Dardelet	1.340	1.208
Average		1.295	1.247
Brass	Coarse	3.104	2.790
	Fine	2.702	2.450
	Dardelet	2.594	2.072
Average		2.800	2.437

It seemed desirable to make a more detailed comparison of the relative effects of load and stretch in comparing static work with impact work. For this purpose, for each material, type of thread, and bolt diameter an "average static stress" (and an "average impact stress") was computed by dividing the "static specific work" (or "impact specific work") expressed in inch-pounds by the "static effective stretch" (or "impact effective stretch"). If the stretch used were the stretch under load at the time of fracture the "stresses" so computed would be the average of the tensile stresses during the breaking of the bolt. The actual stretches under load at the time of fracture could not be determined in the impact tests so that instead for both static and impact loads the residual stretches of the fractured specimens were used. The stresses thus computed were therefore larger than the actual average of the stresses during the breaking of the bolt. For comparative purposes, however, they gave a measure of the relative strength of the bolts under static and impact loads.

In addition an average "ultimate tensile strength" under static load of the material at the root of the threads was computed by dividing the maximum static load by the area at the root of the threads. Some consistent differences between the three types of thread were

found by comparing the results of these computations. For all materials both the average ultimate tensile strength and the average static stress for the American fine thread were smaller than for the other threads, the difference in the average static stress being in all cases greater than in the ultimate tensile strength.

The average impact stress, however, was for all materials greatest for the American coarse thread and (except for the bronze bolts) least for the Dardelet.

The effective stretch showed nearly the reverse of these relationships. For all materials except brass the static effective stretch was greatest for the American fine thread. For all materials the impact effective stretch was least for the American coarse thread and nearly equal for the American fine and Dardelet, averaging slightly higher for the Dardelet.

The consistency of these differences between the different types of thread indicates that for all the materials the type of thread affects in a consistent way each of the factors, static strength, static stretch, impact strength, and impact stretch upon which the ratio of impact work to static work and the ratio of the impact stretch to static stretch depend. The magnitude of the effect upon each factor, however, is different for each material so that similar consistency is not found in the values given in table 15.

It would be desirable to know definitely how each of these factors is influenced by the details of thread shape, such as depth of notch, angle at root, etc., and how this influence differs for different materials. This would, however, require an extensive further investigation. The present investigation planned primarily to give information about commercial forms of bolts, does not furnish the necessary data.

That in general the impact work is greater than the static work is confirmed by other investigators.

Beyer's [10] data given in his reports of tests on ¼ in. and 5/16 in. diameter cold-drawn steel bolts having U. S. Standard threads and Dardelet threads show that the ratio of impact to static work ranges from 0.82 to 2.31, the average ratio being 1.45. This wide range of ratios is probably due to the erratic behavior of cold-drawn steel in impact.

Russell [11] found from transverse tests of rectangular cast-iron bars that the impact work was almost 44 percent greater than the static work. He suggests that this difference may be due to the increase of temperature under impact.

In another study, Russell [12] tested four tensile impact specimens and two static tensile specimens cut from flat bars of different thicknesses. The reduced section was 1 in. in length and ¼ in. in width, while the thickness varied. The impact work was greater than the static work as follows: Norway iron, 23.9 percent; Tennessee charcoal iron, 20.9 percent; Tennessee common iron, 39.4 percent; and soft Bessemer tire steel, 29.6 percent.

[10] Report no. 2162 (Nov. 1929) and no. 2162A (Feb. 1930), Comparative Shock Resistance of Standard V Thread and Nut Connections and Dardelet Thread and Nut Connections. Columbia University (New York, N. Y.). Report no. 2207 (June 1930), Effect of Length of Thread Exposure upon the Static Tensile Strength and Energy to Rupture of Standard V and Dardelet Thread and Nut Connections. Columbia University (New York, N. Y.).

[11] *Experiments with a new machine for testing materials under impact,* Proc. Am. Soc. Civil Engrs. **39**, 237 (1898).

[12] *Tension impact tests of rolled steel.* Eng. News **45**, (no. 1) 14 (Jan. 3, 1901).

Hatt [13] found that there was not much difference between the tensile impact work and the tensile static work for most metals but that the impact work was somewhat greater for steel castings.

Blount, Kirkaldy, and Sanky [14] concluded from their tests "that the energy absorbed per cubic inch in the impact-tensile test is considerably greater than in the static-tensile test, the ratio being approximately 1.6, which may be due to the suddenness of the action".

Cornu-Thenard [15] in 1919 made transverse tests of specimens 10 by 10 by 53.3 mm notched in accordance with the recommendations made at the Copenhagen Congress of the International Association for Testing Materials. For the shock tests the speed was 5.30 mps and for the static tests 60 mph. His results are as follows:

Steel	Ratio of impact to static work
Extra soft	1. 22
Half-hard, nickel, heat treated	1. 01
Hard, chromium-nickel, heat treated	1. 11
Hard, chromium-nickel, annealed	1. 04
Extra soft, coarse grained structure	. 28

They show that except for very soft steel the impact work is greater than the static work. The difference for the chromium-nickel steel, heat treated, is about one-half that for our bolt specimens.

McAdam [16] in discussing the relation of the impact work to the static work refers to the work of other investigators and summarizes the work done on notched specimens at the United States Naval Experiment Station. Carbon and alloy steels were used. The ratio of the impact work to the static work varied from 1.03 to 1.72, the average being 1.36. He concludes that the available data indicate that the impact work, in general, is greater than the static work, because usually the stretch and reduction of area for impact specimens are greater than for static specimens.

Nisley,[17] at Watertown Arsenal, investigated the relation between the impact and static tensile works using specimens machined to a diameter of 0.543 in. for a distance of 2 in. The investigation was limited to one steel, a carbon content 0.39 percent, and six different heat treatments. Five specimens of each heat treatment were tested in the same Charpy impact machine as used in our tests of bolts, and five specimens of each heat treatment were tested in static tension.

The average ratio of impact work to static work varied from 1.63 for heat treatment "A" (6 hr at 1,100° C and furnace cooled) to 1.31 for heat treatment "E" (1 hr at 850° C and water quenched, drawn 1 hr at 600 °C and air cooled).

Petrenko [18] made transverse tests on notched specimens, using a Humfrey machine for the static tests, and an Izod machine for the impact tests. In each of these machines the specimen is fixed at one end and the force applied at the other end. Four notches having different dimensions were used.

Although Petrenko's numerical values do not agree closely with our results on bolts, as is to be expected because of the differences in

[13] *Tensile impact tests of metals*, Proc. Am. Soc. Testing Materials, **4**, 281 (1904).
[14] *Comparison of the tensile, impact-tensile, and repeated-bending methods of testing steel*, Proc. Inst. Mech. Engrs. **1-2**, 715 (1910).
[15] *Sur les essais des flexion par choc de barreaux entaillés*, Comp. Rend. Acad. d Sci. **168**, 1315 (June 30, 1919).
[16] *Impact tests of metals*. Proc. Am. Soc. Testing Materials, **22**, part II, 37 (1922).
[17] *The relation between the dynamic and the static tensile tests*, Army Ordnance, **20**, 88–93 (Sept., Oct., 1923).
[18] *Comparative slow bend and impact notched bar tests on some metals*, BS Tech. Pap. **19**, (1924–25) T289.

the specimens and the method of applying the loads, the high ratios, particularly for brass, in our tests of bolts are in general agreement with his results.

A very careful and complete investigation of the relation between static work and impact work was made by H. C. Mann at Watertown Arsenal.[19] The specimens were machined to a diameter of 0.5 in. for a distance of 2.5 in. The shoulder fillets had a radius of 0.062 in. Three specimens of each material and heat treatment were tested in the Charpy impact machine having a capacity of 2,170 ft-lb, and three of each in static tension. In the static test the stretch up to point of rupture was measured between the shoulders, using a dial micrometer. The chemical compositions of the steels are given in table 16 and the results of the impact and the static tensile tests are given in table 17.

TABLE 16.—*Chemical composition of steels used in impact and static tensile tests from Watertown Arsenal Experimental Report no. 319*

Steel	Chemical compositions							
	C	Mn	P	S	Si	Ni	Cr	V
	Percent	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*
K	0.38	0.18	0.018	0.022	1.460	23.75	7.08	.....
O	.11	.35	.019	.022	.210	3.76	1.39	.....
P	.39	.46	.013	.025	.220	3.44	1.58	.....
R	.22	.52	.010	.021	.235	.09	.91	0.15
T	.23	.75	.010	.031	.410	Trace	.81	.....
U	.44	.71	.023	.021	.455	.24	.77	.....
X	.49	.34	.020	.022	.165	1.76	.99	.....

TABLE 17.—*Results of impact and static tensile tests from Watertown Arsenal Experimental Report no. 319*

Steel		Microstructure after heat treatment	Brinell number	Impact work, average	Static work, average	*Ratio*
Symbol	Description					Impact work / Static work
				ft-lb	ft-lb	
K	"Stainless"	Austenitic	207	1,250	983	1.27
		do	196	1,190	933	1.28
O	SAE no. 3,312	Sorbo-pearlitic	187	790	584	1.35
	High nickel-chromium	Sorbitic	196	940	669	1.40
P	SAE no. 3,340 nickel-chromium.	do	269	953	718	1.33
R	SAE no. 6,120	Sorbo-pearlitic	149	937	631	1.48
	Chromium-vanadium	do	217	924	656	1.41
T	SAE no. 5,120	do	143	924	650	1.42
	Case hardening steel	Sorbitic	196	1,023	743	1.38
U	SAE no. 5,140	do	207	947	694	1.36
		do	207	980	709	1.38
X	SAE no. 3,250	Troosto-Sorbitic	228	1,003	750	1.34
		Sorbitic	228	993	735	1.35

The report states that "The effect of volume has been investigated sufficiently to establish the fact that specimens of different volume but of the same microstructure absorb an amount of the energy of impact in proportion to the volume."

[19] Watertown Arsenal Experimental Report no. 319.

VIII. CONCLUSIONS

Impact and static tensile tests were made on bolts with nuts having American National coarse threads, American National fine threads, and Dardelet threads. The nominal diameters of the bolts were ⅜, ½, ⅝, and ¾ in. The materials were heat-treated chromium-nickel steel, cold-rolled steel (Bessemer screw stock), monel metal, bronze (copper-silicon-manganese alloy), and brass. The length of each bolt from the head to the bearing face of the nut was five times the nominal diameter. The exposed threads on the bolt extended from the bearing face of the nut toward the head for one diameter. Four similar specimens of each material, diameter, and thread were tested under single-blow impact loading and two under static loading. The total number of specimens was 360. The following conclusions were drawn from the results of the tests:

1. In the impact tests two chromium-nickel bolts with American National fine threads failed by stripping the thread. Over one-fourth of the cold-rolled steel bolts with American National threads, both coarse and fine, had "brittle" failures. These anomalous failures gave erratic and low impact values. Otherwise, similar bolts having Dardelet threads gave no such erratic results.

2. Except for the brass bolts and those which showed stripped threads or brittle failures, the work per unit volume required to rupture the bolts either under impact loading or under static loading was for each material and for each type of thread approximately independent of the diameter of the bolts. There was, however, an indication that the work per unit volume was dependent somewhat upon the ratio of minor to major diameter of the thread.

3. Except for the brass bolts and those which had stripped threads or brittle failures, the stretch in the one diameter of exposed threads either under impact loading or under static loading was for each material and each type of thread approximately independent of the diameter of the bolts.

4. In all cases the impact works for bolts having American National coarse threads were less than the impact works for bolts of the same size and material having American National fine threads.

5. Except for the brass bolts and those which showed stripped threads or brittle failures, the impact works for bolts having American National fine threads were approximately the same as the impact works for bolts of the same size and material having Dardelet threads. The impact works for brass bolts having American National fine threads were on the average 12 percent less than the impact works for brass bolts of the same size having Dardelet threads.

6. In all cases the impact works for bolts having Dardelet threads were much greater than the impact works for bolts of the same size and material having American National coarse threads.

7. In all cases the static works for bolts having American National coarse threads were less than the static works for bolts of the same size and material having American National fine threads.

8. Except for the brass bolts and those which showed stripped threads or brittle failures, the static works for bolts having American National fine threads were approximately the same as the static works for bolts of the same size and material having Dardelet threads. For the brass bolts the static works for bolts having American Na-

tional fine threads were on the average approximately 26 percent lower than the static works for bolts of the same size having Dardelet threads.

9. In all cases the static works for bolts having Dardelet threads were greater than the static works for bolts of the same size and material having American National coarse threads.

10. In all cases the maximum static loads for bolts having American National coarse threads were less than the maximum static loads for bolts of the same size and material having American National fine threads.

11. In all cases the maximum static loads for bolts having American National fine threads were approximately the same as the maximum static loads for bolts of the same size and material having Dardelet threads.

12. In all cases the maximum static loads for bolts having Dardelet threads were greater than the maximum static loads for bolts of the same size and material having American National coarse threads.

13. For bolts of the same size and having the same threads the static-bolt efficiencies, defined as the ratio of the maximum static load for the bolt to the load computed by multiplying the cross-sectional area of the shank by the ultimate tensile strength of the material, were approximately the same for all of the materials.

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U. S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS

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A METHOD FOR DETERMINING STRESSES IN A NONROTATING PROPELLER BLADE VIBRATING WITH A NATURAL FREQUENCY

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ABSTRACT

Propeller failures in flight generally have the appearance of fatigue fractures. This points to resonant vibrations setting up excessive alternating stresses as a probable cause of failure. The present paper describes a method for measuring and for calculating such alternating stresses for the case of a nonrotating propeller vibrating in resonance with an alternating torque applied to its shaft. A complete picture of the stress distribution in such a propeller was obtained by measuring the longitudinal and transverse strain amplitudes at various points, and calculating the stresses from these measurements, assuming a condition of plane stress at the surface of the blade. Stress distributions were obtained for a duralumin blade vibrating with its fundamental mode and also for the same blade vibrating with its second harmonic mode (with a node near the tip). The measured stress distributions and frequencies for the two modes were checked by the stresses and frequencies calculated from the theory of vibrating beams of variable section. The effect of restraint at the hub on frequency and on stress distribution was also investigated theoretically, and it was found that the degree of restraint at the hub affected the stress distribution very little, but that it had considerable effect on the natural frequency. A further check on the measurements was made by noting that artificially produced fatigue failures in eight nonrotating blades vibrating with their fundamental mode occurred in each case at a section where the stresses were within a few percent of the maximum stress measured.

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I. STATEMENT OF PROBLEM

An investigation of the vibration of aircraft propeller blades, having as its ultimate aim the diagnosis of the failure of propellers in flight, is being carried out at the National Bureau of Standards. Failures occur in most cases either close to the hub or a few inches from the tip of the blade. They have the characteristic appearance of fatigue fractures, thus pointing to alternating stresses as the cause. The source of such alternating stresses is not definitely known; but it is known that alternating stresses occur whenever a periodically varying torque is applied to the propeller shaft or whenever periodic impulses are applied to the blades themselves. It is also known that periodic forces in resonance with one of the natural frequencies of the propeller-shaft system may set up propeller vibrations of relatively large amplitude.

It seemed logical to start the investigation with propellers vibrating without rotation in order to get a clear picture of their behavior under these relatively simple conditions before going to the more complex ones of the rotating propeller. There were good reasons to think that such a preliminary investigation would result in the development of a technique and in the assembly of data that would later be of value in considering propellers in flight. For example, it has been demonstrated that the natural frequency of the fundamental mode of a rotating propeller can be estimated with some degree of accuracy in a simple manner from the natural frequencies without rotation.[1]

Considerations similar to these suggested the following line of attack on the problem:

1. Develop a method for applying periodic impulses of controllable frequency and amplitude to nonrotating propeller blades so as to set up resonant vibrations of sufficient amplitude to cause failure, if possible.
2. Develop a method for measuring the stresses set up in the vibrating blade and, if possible, obtain values at the points of failure for the stresses just preceding failure.
3. Extend these methods to propeller blades in rotation.

The present paper will deal principally with the second part of the research program, that is, with a description of a method developed for determining the stresses in a nonrotating propeller blade vibrating with a natural frequency.

II. EXPERIMENTAL PROCEDURE

1. EXCITATION OF VIBRATION

The first part of the research program has already been described in an article by L. B. Tuckerman, H. L. Dryden, and H. B. Brooks.[2] The application of the method there described to the present problem will be clear after an examination of the photograph, figure 1. G in this figure is a 27-hp motor-generator set which is able to supply nearly sinusoidal alternating current in a range of frequencies from 10 to 180 cycles/second. This alternating current is fed into the armature of the 15-hp motor, M; the field coils of this motor are excited by direct current so that sinusoidal impulses of torque are set up in the shaft of the motor with an amplitude that is nearly proportional (up to saturation of the magnetic field) to the product of the field current

[1] F. Liebers. Jahrb. Deut. Versuchsanstalt Luftfahrt **2**, 20-28 (1932).
[2] BS J. Research **10**, 659 (1933) RP556.

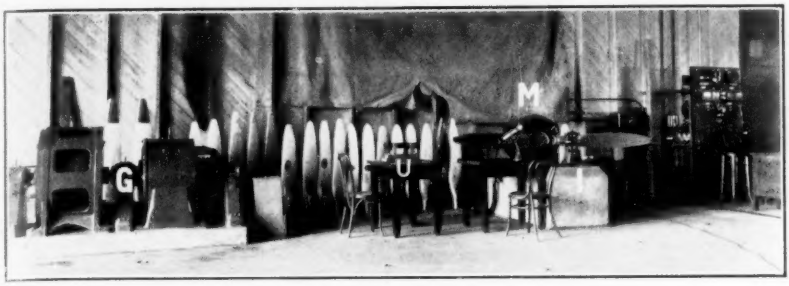

FIGURE 1.—*Propeller vibration apparatus.*

Over-all view, final set up (G, 27-hp variable-frequency motor-generator set; U, telescope for observing tip amplitude; M, 15-hp direct-current motor; I, outboard bearing for motor shaft).

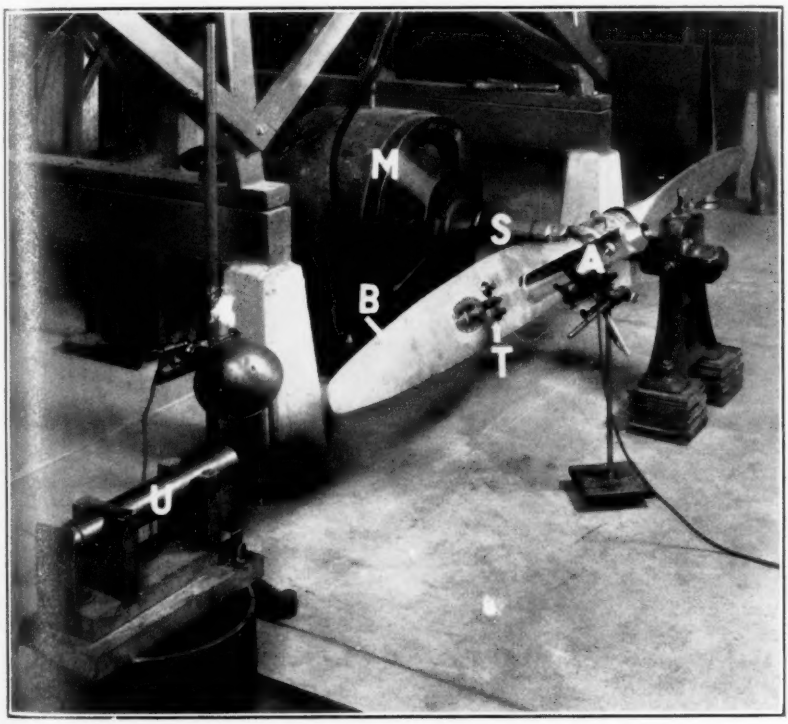

FIGURE 2.—*Propeller vibration apparatus, close up.*

Initial set up (U, telescope for observing tip amplitude; M, 7½-hp direct-current motor; S, motor shaft; B, propeller blade; T, Tuckerman optical strain gage; A, Tuckerman autocollimator).

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and the armature current, and with a frequency equal to the frequency of the armature current. These torsional impulses are transmitted to the propeller through the shaft of the motor. The stand, I, in front of the propeller carries a 4-screw bearing for holding the forward tip of the shaft. This bearing was used to prevent excessive vibration of the shaft as a cantilever beam heavily loaded at the end.

2. AMPLITUDE OF VIBRATION

The amplitude of vibration of the propeller blade was observed by measuring with the telescope, U, the motion of an illuminated point at one of the two tips (figs. 1, 2; 2 is an earlier assembly). The telescope had a special reticule on which a sheet of cross-section paper had been photographed. The motion of the vibrating point appears as a bright line magnified 10 times on the cross-section paper.

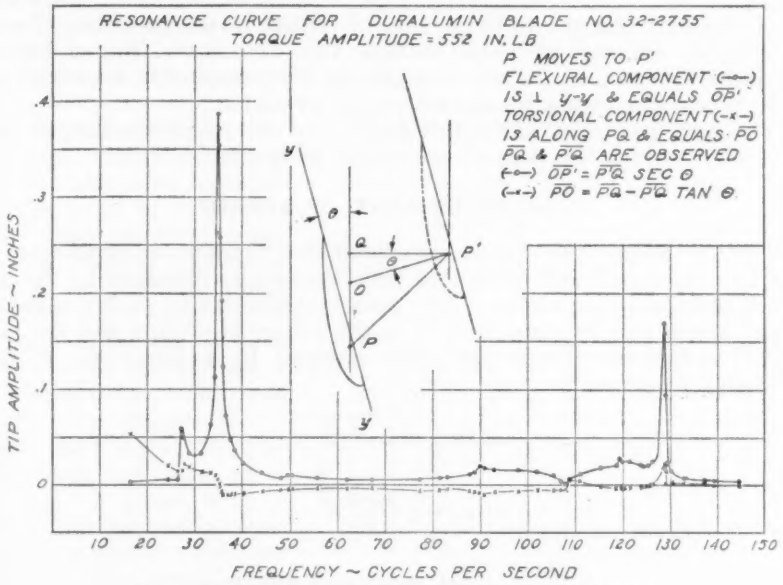
RESONANCE CURVE FOR DURALUMIN BLADE NO. 32-2755
TORQUE AMPLITUDE = 552 IN. LB
P MOVES TO P'
FLEXURAL COMPONENT (—o—)
IS ⊥ y-y & EQUALS OP'
TORSIONAL COMPONENT (-x-)
IS ALONG PQ & EQUALS PO
PQ & P'Q ARE OBSERVED
(—o—) OP' = P'Q SEC θ
(-x-) PO = PQ - P'Q TAN θ
TIP AMPLITUDE ~ INCHES
FREQUENCY ~ CYCLES PER SECOND

FIGURE 3.—*Resonance curve for a duralumin blade.*

The amplitude can be measured to about 0.01 in. by reading the end points of this line. The plane of motion follows directly from the angle which the bright line makes with the axes of the cross-section paper. The magnitude of the motion may be regarded as the vector sum of two motions, one in the plane of rotation of the propeller (OP, fig. 3), the other normal to the plane of the blade at the tip (OP', fig. 3). The components OP, OP' are obtained directly from the components of motion, PQ in the plane of rotation of the blade and QP' normal to that plane, by setting one axis of the cross-section paper parallel to the plane of rotation of the blade. It is then easily shown from the sketch in figure 3 that

$$\begin{aligned} OP &= PQ - P'Q \tan\theta \\ OP' &= QP' \sec\theta \end{aligned} \qquad (1)$$

where θ is the angle which the plane of the blade at the tip makes with the plane of rotation of the propeller.

In this way the torsional motion OP of the shaft may be separated from the flexural motion OP′ of the blade, and separate resonance curves obtained for each motion. Figure 3 shows the result for a duralumin blade of common design (no. 32–2755). The amplitude at the tip for an armature current of 15.0 amp corresponding to a torque amplitude of 552 in-lb is here plotted against the frequency for a range of frequencies from about 15 to 140 cycles/sec. Two resonance peaks are noticeable; the first one about 35 cycles/sec corresponding to a vibration of each blade similar to a cantilever beam fixed at one end, and the second one around 130 cycles/sec corresponding to a vibration of the blade as a cantilever beam with a node near the tip of the blade. It is seen at once from figure 3 that by far the largest amplitudes were observed when the propeller was excited with its fundamental cantilever beam frequency at around 35 cycles/sec. This amplitude could in fact be increased to a value that was high enough to cause fatigue failure in the propeller after a few hours running (see also section VI). No such fatigue failure could be obtained at the other resonance frequency even by vibrating the propeller with the maximum power available.

The next step after the determination of the resonance frequencies was to determine the stresses associated with them.

3. MEASUREMENT OF STRAIN

The curvature of the blade is so small that the stresses at its surface can be sufficiently well computed from the strains obtained by assuming a state of plane stress. The principal stresses in such a state of plane stress can be computed from the principal strains and the two elastic constants of the propeller material by making use of the following relations: [3]

$$\sigma_x=\frac{\epsilon_x+\mu\epsilon_y}{1-\mu^2}E \qquad (2)$$

$$\sigma_y=\frac{\epsilon_y+\mu\epsilon_x}{1-\mu^2}E$$

where:

σ_x = principal stress in x-direction.
σ_y = principal stress in y-direction.
ϵ_x = principal strain in x-direction.
ϵ_y = principal strain in y-direction.
E = Young's modulus of material.
μ = Poisson's ratio of material.

The principal strains ϵ_x, ϵ_y may be considered as the semiaxes of a strain ellipse. The determination of this strain ellipse at a given point on the propeller blade requires the observation of strains in four different directions about the point. Three would suffice if we were dealing with static strain; one more is needed if the strains vary as a simple harmonic motion, whose amplitude alone can be measured, in order to determine the phase relation between ϵ_x and ϵ_y. It was found experimentally that the motion of any point on the propeller blade when vibrating in resonance with its fundamental frequency was close to such a simple sine motion.

[3] S. Timoshenko. Strength of Materials. **1**, 58. (D. Van Nostrand Co., New York, 1930.)

Knowing the strains in any four directions about a point, the principal strains can be derived by a geometrical construction.[4]

The experimental procedure for obtaining the strain readings was as follows. A 2-inch Tuckerman optical strain gage (T, fig. 2) was held firmly against the blade surface by two vacuum cups, with its knife-edge and lozenge each 1 inch from the point studied. The blade was caused to vibrate at its resonance frequency and the amplitude at the tip was observed through the telescope, U, in the manner described above. The average strain amplitude over a 2-inch span was read off directly by using the calibrated Tuckerman autocollimator, A, with the "dumbbell" reticule especially designed for the measurement of dynamic strains.[5]

It should be noted in this connection that the usual 30° knife-edge was replaced by a 90° knife-edge to minimize the error introduced by the bending of the knife-edges by inertia forces resisting the acceleration of the gage in its simple harmonic motion. It can be shown theoretically that the error from this source may be eliminated by reading the gage in one position, turning it 180°, reading it again, and taking the average of the two readings, provided the deflection due to inertia forces is smaller than the extension to be measured. This was verified experimentally by noting that the average of two strain readings at a given point taken with a 30° knife-edge was in close agreement with the average obtained with the 90° knife-edge, although the difference between individual readings (about 40 percent for the 30° knife-edge) was about five times as large in the first case as in the second. The difference in the two readings using the heavy knife-edge was negligible for most points on a blade vibrating with its fundamental frequency.

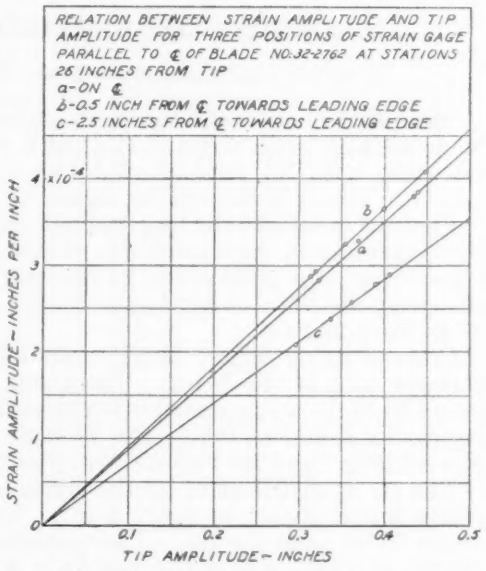

FIGURE 4.—*Relation between strain amplitude and tip amplitude.*

The strain for a given position of the strain gage was obtained for at least four different tip amplitudes ranging from 0.05 to 0.5 in., the tip amplitudes [6] being increased by increasing the armature current of the driving motor. It was found that the strain was proportional to the tip deflection, the observations lying close to a straight line when strain amplitude was plotted against tip amplitude (see fig. 4). The slope of the straight line faired through the observed points gives the strain amplitude for 1 in. tip amplitude. The propor-

[4] W. R. Osgood and R. G. Sturm. BS J. Research **10**, 685 (1933) RP559.
[5] George K. Burgess. Paper 335, pp. 33-36. World Eng. Cong., Tokyo (Nov. 1929).
[6] The term amplitude as used here designates one-half the total excursion of the vibrating mark.

tionality between strain amplitude and tip amplitude was verified at a point near the hub up to a tip amplitude of about 1½ in.; that is, up to amplitudes approaching those at which fatigue failure occurred.

Strains per inch of tip amplitude were determined in this manner in four directions about several points, and the principal strains were computed, using the method described by Osgood and Sturm.[7] The two principal strains were found parallel and at right angles, respectively, to an approximate center line which was constructed by connecting points that were equidistant from the leading and the trailing edge of the blade. Hence it was thought sufficient to take two gage readings for each point studied, one parallel to this center line and the other normal to it, and to assume that the strains thus observed were equal to the principal strains and could, therefore, be substituted in the expressions eq 2 for the principal stresses.

III. ANALYSIS OF EXPERIMENTAL RESULTS

STRESS DISTRIBUTION FOR FUNDAMENTAL MODE

Some of the results of a thorough exploration of a 47.6 in. duralumin blade set at a pitch angle of 19.6° at a point 5.5 in. from the tip and vibrating with its fundamental frequency (34.8 cycles/sec in this case) are shown in the curves, figure 5.

Figure 5 (a) shows the longitudinal and the transverse strains along the center line on the convex side of the blade. The strain ϵ_x becomes a maximum at a point about 26 in. from the tip. Figure 5 (b) shows the strain distribution across the convex side of the blade at a section 26 in. from the tip.

Corresponding strain distribution curves were obtained for the concave side of the blade. The stress distribution was then computed from the strain distribution by substituting in eq 2 the observed strains for ϵ_x and ϵ_y, taking the Young's modulus of the material as $E=10^7$ lb/in.^{2} and its Poisson's ratio as 0.3.

The stress distribution on the concave side of the blade along the center line is shown in figure 5 (c). The stress is largest at a point about 27 in. from the tip. The transverse stress is compressive near the tip and tensile near the hub; it is in no case more than one-third the longitudinal stress. This behavior may be explained qualitatively from the shape of the blade, remembering that the direction of the principal longitudinal stress near the edges of the blade must be nearly parallel to the respective edges. The edges tend to converge towards the tip, and the corresponding stresses make an angle with each other, thus throwing the material between them into compression in a transverse direction.

Figure 5 (d) shows the stress distribution on the concave side of the blade at a section 26 in. from the tip. The transverse stress is nearly 0, as it would be for a rectangular beam. The longitudinal stress increases steadily in passing from the trailing edge toward the leading edge up to a point near the leading edge; this behavior may be explained by the change in thickness of section and consequent change in the distance from the neutral surface to the extreme fiber in proceeding from one edge of the blade to the other.

[7] W. R. Osgood and R. G. Sturm. BS J. Research **10**, 685 (1933) RP559.

The stress distribution along the centerline on the convex side of the blade is shown in figure 5 (e). This is of the same general nature as the corresponding stress distribution on the concave side (fig. 5 (c)).

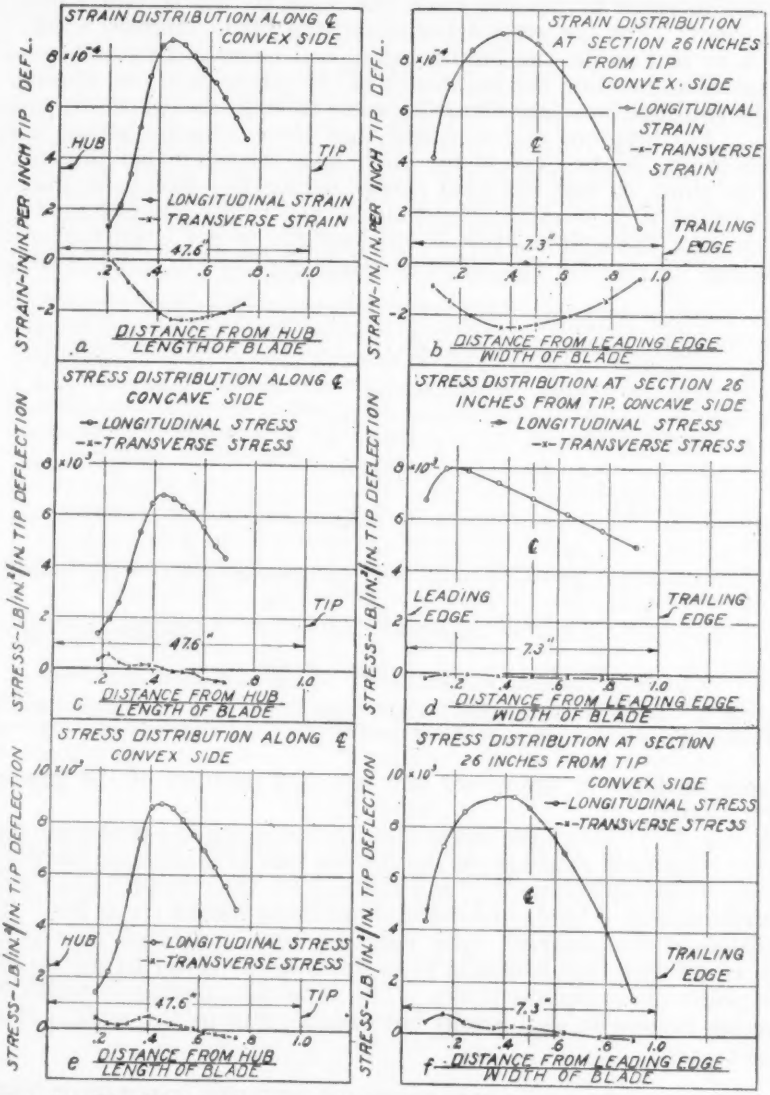

FIGURE 5.—*Distribution of stress and strain at extreme fiber for duralumin blade type 32, vibrating with fundamental mode.*

Figure 5 (f) shows the stress distribution on the convex side at a section 26 in. from the tip. The longitudinal stress is higher than on the concave side, corresponding to the greater distance to the extreme fiber. The transverse stress is no longer zero; this is not surprising since the convex side is not so nearly plane as the concave side.

2. STRESS DISTRIBUTION FOR SECOND HARMONIC MODE

All the stress distributions described so far were observed in a blade vibrating with its fundamental frequency. The experimental difficulties increase in passing from this case to the second harmonic frequency. For the second harmonic, the blade vibrates about 3.7 times as fast as for the fundamental, and consequently the accelerating forces bending the knife-edge of the strain gage and tending to shake the gage loose are about 14 times as large at a given tip amplitude. Furthermore, it was found that the maximum stress in this mode occurs in the third of the blade near the tip rather than in the middle third, so that the most careful exploration had to be made in that region with its large amplitude normal to the plane of the blade. In spite of these difficulties it was possible to get consistent strain readings by running the blade at relatively low tip amplitudes and

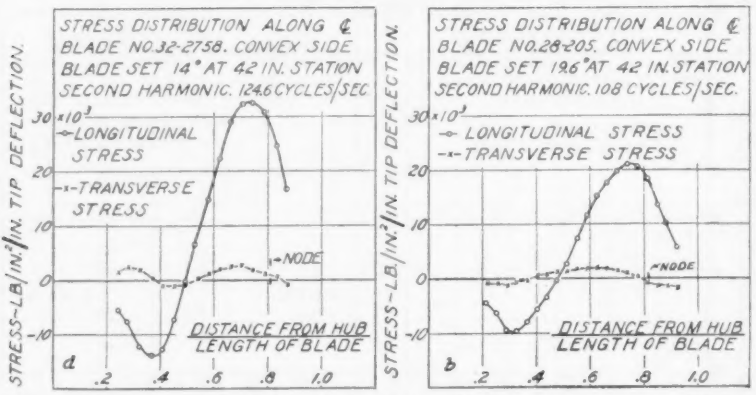

FIGURE 6.—(a) *Stress distribution for blade type 32 vibrating with second harmonic mode;* (b) *stress distribution for blade type 28 vibrating with second harmonic mode.*

taking readings in the direct and reversed position of the gage for each setting.

Figure 6 (a) shows the stress distribution along the centerline computed from such readings on the convex side of a blade of the design so far considered (type 32), and figure 6 (b) the corresponding stress distribution on a blade of somewhat different design (type 28).[8] The position of the observed node near the tip is indicated on the two figures; in each case it is seen to be appreciably closer to the tip than the point of maximum stress. The inflection points at which the stresses reverse in sign are in both cases close to the middle of the blade.

IV. THEORETICAL ANALYSIS OF STRESS DISTRIBUTION IN A PROPELLER BLADE VIBRATING WITHOUT ROTATION

It was deemed worth while as a check on both the technique used and the experimental results obtained to compare these results with those given by a theoretical analysis. Hansen and Mesmer[9] calcu-

[8] The scatter of observed strain amplitudes plotted against tip amplitudes for this second blade was reduced by replacing the original Tuckerman gage by one of special design that was lightened as much as possible so as to minimize inertia forces.

[9] Z. Flugtech. Motorluftschiffahrt **24**, 298 (1933).

lated the stress distribution in a propeller blade vibrating without rotation by setting up the equation of motion of the blade, solving the equation for a natural mode, and from the mode calculating the stress at any point. A brief description of the analytical procedure used is given below.

1. METHOD

(a) EQUATION OF MOTION OF BLADE

The exact equations of motion of a beam with an unsymmetrical cross section whose area, moments of inertia, and principal axes of inertia vary along its length would be extremely complicated, even if they could be written down. For this reason it is customary [10] to treat the problem by an approximate method, assuming that the motion of the blade is adequately represented by the motion of a beam with symmetrical cross sections, whose areas and principal moments of inertia are equal to those of the propeller blade, and whose principal axes of inertia remain parallel throughout the length of the beam. This approximate treatment cannot reproduce torsional vibrations of the blade, nor give any close approximation to the natural vibrations of high order. It does give a useful approximation to the flexural vibrations of lower frequency.

Since the longitudinal dimensions of the blade are large compared with the lateral dimensions, it is further customary to neglect the effect of shear and rotational inertia on the motion of the blade. For a propeller blade of usual design this introduces only small errors [11] at the lower frequencies, and it is convenient in that it permits one to use the ordinary beam theory in setting up the differential equation of motion of the blade.

The motion of each particle in this idealized blade will be sinusoidal if, as in the present case, the blade is vibrating under the action of a sinusoidal exciting force or moment applied at the hub in a principal plane of inertia. One may then express the displacement at any section by eq 3:

$$z = X \sin pt \tag{3}$$

where $p/2\pi$ is the frequency of the exciting force or moment in cycles/second and X is the amplitude of displacement at a distance x from the center of the hub. One has a complete picture of the motion if both X and p are known.

The differential equation which X must satisfy is obtained by substituting eq 3 in the equation:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2z}{dx^2}\right) = -\rho A\frac{d^2z}{dt^2} \tag{4}$$

expressing the equilibrium of shear forces and inertia forces in a transverse direction; EI denotes the flexural rigidity of the section of the blade considered and ρA the mass per unit length at that same section. Inserting eq 3 in eq 4 yields:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2X}{dx^2}\right) = \rho A p^2 X \tag{5}$$

[10] See for instance Griffith. Advisory Comm. Aeronaut. (Gt. Brit.) R. & M. no. 451 (1918).
[11] S. Timoshenko . Vibration Problems in Engineering, p. 231. (D .Van Nostrand Co., New York, 1928.)

The solution of this differential equation is simple in the case of a beam in which both the coefficients EI and A are constant. But in the case of a propeller these coefficients are not even approximately constant. This is brought out clearly by the curves of figure 7 (a) and 7 (b), respectively, which show the distribution of $\frac{\sqrt{A}}{L}$ and $\frac{\sqrt[4]{I}}{L}$ plotted against the relative distance x/L from the center of the hub for blade 32–2758; this blade is of the same design (type 32) as blade 32–2762. Both quantities decrease rapidly from the hub to the tip; A and I will decrease even more rapidly since they are proportional to the square and the fourth power of these quantities, respectively. The distribution of both A and I was obtained from direct measurements of the section of the blade at 15 sections along its length of 47.6 in.

Neither A nor I vary with x in a simple way. There seemed little hope, therefore, of obtaining for them an analytical expression that

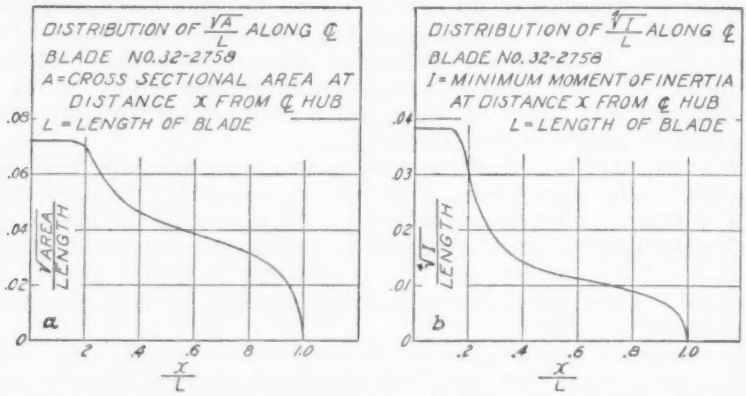

FIGURE 7.—*Variation of cross-sectional area* (a), *and of moment of inertia* (b) *along blade.*

would give to the differential eq 5 an integrable form. Hence, it was decided to make use of the numerical method of solution that has been applied to a number of propellers by Hansen and Mesmer [12]; it is the mathematical counterpart of a well-established graphical method for the computation of natural modes and frequencies of rotating shafts of varying section.[13]

The formula used by Hansen and Mesmer [14] to compute X and p for the vibration of a bar may be derived as follows:

Replace the independent variable x by the ratio:

$$\xi=\frac{x}{L} \tag{6}$$

so that eq 5 takes on the form:

$$\frac{d^2}{d\xi^2}\left(\frac{I}{L^4}\frac{d^2X}{d\xi^2}\right)=\rho\frac{p^2L^2}{E}\frac{A}{L^2}X \tag{7}$$

where I/L^4 and A/L^2 are both known functions of ξ and where ξ varies from 0 to 1 as one passes from the point of clamping to the tip of the blade.

[12] Z. Flugtech. Motorluftschiffahrt, **24**, 298 (1933).
[13] S. Timoshenko. Vibration Problems in Engineering, p. 231 (D. Van Nostrand Co., New York, 1928).
[14] Hansen and Mesmer limited their discussion to a clamped-free bar.

X is a function of ξ alone independent of the time t, of the form

$$\frac{X}{d}=f(\xi)$$

where d is the amplitude of displacement at the tip of the blade ($\xi=1$). For each value of p, $f(\xi)$ is a definite function of ξ depending only on the type (force or moment) and the frequency of the excitation, the dimensions of the bar, and the end conditions.

The present work is concerned primarily with conditions of resonance. At resonance the exciting force or moment becomes negligible in comparison with the inertial reactions of the vibrating blade.

For the clamped-free bar one can, therefore, assume for the boundary conditions at the tip end ($\xi=1$), zero shear, zero bending moment; at the hub end ($\xi=0$), zero slope, zero deflection.

$$\left.\begin{aligned}\frac{d}{d\xi}\left(\frac{I}{L^4}\frac{d^2X}{d\xi^2}\right)=0,\ \frac{I}{L^4}\frac{d^2X}{d\xi^2}=0,\ \text{at } \xi=1\\ \frac{dX}{d\xi}=0,\ X=0,\ \text{at } \xi=0\end{aligned}\right\} \tag{8}$$

Any value of X as a function of ξ with the corresponding value of p, that satisfies both eq 7 and 8 represents a "natural" mode of vibration of the blade. An expression for X (ξ) is obtained by integrating eq 7 four times in succession, determining each time the constant of integration from one of the four boundary conditions eq 8. This leads to the following integral equation for the natural mode of a clamped-free bar:

$$X=\frac{\rho p^2 L^2}{E}\int_0^\xi\int_0^\xi\frac{L^4}{I}\int_1^\xi\int_1^\xi\frac{A}{L^2}X\quad d\xi d\xi d\xi d\xi \tag{9}$$

(b) NUMERICAL SOLUTION OF EQUATIONS OF MOTION

The final deflection curve X (ξ) on the left of eq 9 must have the same shape as the assumed deflection curve X (ξ) in the integrand on the right if the latter describes a possible mode of vibration of the blade. To test this condition in a given case one must compute two double integrals of the type:

$$\left.\begin{aligned}F=\int_1^\xi\int_1^\xi\varphi(\xi)d\xi d\xi\\ G=\int_0^\xi\int_0^\xi\psi(\xi)d\xi d\xi\end{aligned}\right\} \tag{10}$$

where φ (ξ), ψ (ξ) are known functions of ξ; in this particular case they are:

$$\varphi(\xi)=\frac{A}{L^2}X,\ \psi(\xi)=\frac{L^4}{I}F$$

F may be converted into the form G by replacing ξ by $1-\xi$, so that only the second form (G) need be considered. Hansen and Mesmer [15] use a convenient method for integrating this second form numerically. It consists in subdividing the interval from $\xi=0$ to $\xi=1$ into n equal

[15] Z. Flugtech. Motorluftschiffahrt 24, 298 (1933).

intervals of width $h=1/n$ and tabulating the values of $\psi(\xi)$ for the boundaries of these intervals:

$$\psi(0),\ \psi\left(\frac{1}{n}\right),\ \psi\left(\frac{2}{n}\right),\ \ldots,\psi\left(\frac{i}{n}\right),\ \psi\left(\frac{i+1}{n}\right),\ \ldots,$$

The interval spacing h is chosen small enough so that the integrand $\psi(\xi)$ in the interval $\frac{i}{n}\leq\xi\leq\frac{i+2}{n}$ $(i=0, 1, 2, \ldots, n-2)$ may be represented accurately by a parabola:

$$\psi(\xi)=c_{0i}+c_{1i}\xi+c_{2i}\xi^2 \tag{11}$$

The 3 constants of this parabola are then determined from the condition that $\psi(\xi)$ must go through the 3 points $\psi\left(\frac{i}{n}\right),\ \psi\left(\frac{i+1}{n}\right),\psi\left(\frac{i+2}{n}\right)$.

The value of $G(\xi)$ at any interval may now be found by substituting eq 11 in eq 10 and integrating twice with respect to ξ. This leads to the following formulas:

$$\left.\begin{aligned} G_0&=0\\ G_1&=\frac{1}{12n^2}(3.5\psi_0+3\psi_1-0.5\psi_2)\\ G_i&=2G_{i-1}-G_{i-2}+\frac{1}{12n^2}(\psi_{i-2}+10\psi_{i-1}+\psi_i)\end{aligned}\right\} \tag{12}$$

where G_0 and ψ_0 denote values of $G(\xi)$ and $\psi(\xi)$ at the origin $(\xi=0)$ and G_i and ψ_i values of the same quantities at the end of the i-th interval $\left(\xi=\frac{i}{n}\right)$. $F(\xi)$ can be computed in the same way, remembering only that the direction of integration must be reversed. This leads to the formulas:

$$\left.\begin{aligned} F_n&=0\\ F_{n-1}&=\frac{1}{12n^2}(3.5\varphi_n+3\varphi_{n-1}-0.5\varphi_{n-2})\\ F_i&=2F_{i+1}-F_{i+2}+\frac{1}{12n^2}(\varphi_{i+2}+10\varphi_{i+1}+\varphi_i)\end{aligned}\right\} \tag{13}$$

where F_i and φ_i denote values of $F(\xi)$ and $\varphi(\xi)$ at the end of the i-th interval $\left(\xi=\frac{i}{n}\right)$.

In the present work it was found convenient to subdivide the length of the blade into 20 equal intervals, thus choosing $n=20$. Table 1 illustrates a convenient arrangement for computing all values of F and G except F_{n-1} and G_1, which were computed directly from the second formulas of eq 12 and eq 13. The table was taken from the computation of the fundamental mode of a blade of type 32 vibrating as a clamped-free bar. Part A shows the computation of the integral F_0, choosing for the mode $X=X_0$ values that are proportional to the deflection curve of the blade under its own weight. φ_i and F_i denote values of $\varphi=\frac{AX_0}{L^2}$ and F at the end of the i-th interval.

Part B of the table is carried out the second double integration, namely, that leading to G as given by eq 10; $\frac{L^4F}{I}$ is substituted for ψ (ξ), where F is taken from the previous table. In the last row of part B is the value of G at the end of each interval. It is, according to eq 9, proportional to the mode X_1 corresponding to an inertia loading produced by the assumed mode X_0. The ratio X_0/G_0, as shown in a separate row below the tables, ranges from 2.12×10^{-3} to 3.56×10^{-3}. This shows that the eq 9 is far from being satisfied. A closer degree of proportionality will be obtained if the process is now repeated starting with a mode proportional to X_1 (which is proportional to G_0) and leading to a mode X_2 proportional to the distribution $G_1(\xi)$ obtained in this second calculation. The ratio X_{m+1}/G_m may be made constant to any degree of accuracy, provided the procedure is repeated a sufficient number of times. In the present work constancy within slide rule error was considered sufficient.

TABLE 1.—*First approximation in computation of fundamental mode duralumin blade, type 32.*

A. Computation of $\frac{E}{\rho p^2 L^2} X_1 = \int_0^\xi \int_0^\xi \frac{L^4}{I} \int_1^\xi \int_1^\xi \frac{A}{L^2} X_0 d\xi d\xi d\xi d\xi$

$F_0(\xi) = \int_1^\xi \int_1^\xi \frac{A}{L^2} X_0 d\xi d\xi = \int_1^\xi \int_1^\xi \varphi d\xi d\xi$

Computation begins at the right column and proceeds toward the left.

ξ	0.	* * * *	0. 20	* * * *	0. 85	0. 90	0. 95	1. 00
(a) i	0.	* * * *	4.	* * * *	17.	18.	19.	20.
(b) X_0	0. 0000	* * * *	4. 787	* * * *	627. 2	720. 5	814. 2	908. 0
(c) $\varphi_i = \frac{A}{L^2} \cdot X_0$	+0. 0000	* * * *	+0. 0233	* * * *	+0. 5332	+0. 4756	+0. 3257	0. 0
(d) 10 φ_{i+1}	+. 0105	* * * *	+. 2961	* * * *	+4. 7560	+3. 2570		
(e) φ_{i+2}	+. 0059	* * * *	+. 0377	* * * *	+. 3257	. 0000		
(f) $2 \cdot 12n^2 F_{i+1}$	+1399. 0000	* * * *	+981. 2000	* * * *	+10. 4200	+1. 4790		
(g) (c)+(d)+(e)+(f)	+1399. 0000	* * * *	+981. 5000	* * * *	+16. 0400	+5. 2120		
(h) $12n^2 F_{i+2}$	647. 0000	* * * *	439. 0000	* * * *	. 7393	0. 0000		
(i) (g)−(h)$=12n^2 F_i$	+752. 0000	* * * *	+542. 5000	* * * *	+15. 3000	+5. 2120	[a]+. 7393	[a]0. 0
(j) $\frac{(i)}{12n^2} = F_i = F_0(\xi)$	0. 1566	* * * *	0. 1131	* * * *	0. 00319	0. 001086	0. 000154	0. 0

[a] Computed by first and second parts of equations 13.

B. Computation of $G_0(\xi) = \int_0^\xi \int_0^\xi F^0 \frac{I}{L^4} d\xi d\xi = \int_0^\xi \int_0^\xi \psi d\xi d\xi$

Computation begins at the left column and proceeds toward the right.

ξ	0.	0. 05	0. 10	0. 15	0. 20	* * * *	0. 95	1. 00
(a) i	0.	1.	2.	3.	4.	* * * *	19.	20.
(c) $\psi i = F_o(\xi) \cdot \frac{I}{L^4}$	+73300.	+68100.	+63100.	+58500.	+117800.	* * * *	+118000.	0.
(d) $10\psi_{i-1}$			+681000.	+631000.	+585000.	* * * *	+3880000.	+118000.
(e) ψ_{i-2}			+73300.	+68100.	+63100.	* * * *	+709000.	+38800.
(f) $2 \cdot 12n^2 G_{i-1}$			+858600.	+3352000.	+7362000.	* * * *	+3186000000.	+364800000.
(g) (c) + (d) + (e) + (f)			1676000.	+4110000.	+8128000.	* * * *	+3191000000.	+365000000.
(h) $12n^2 G_{i-2}$			000.	429300.	1676000.	* * * *	1367000000.	159300000.
(i) (g) − (h) = $12n^2 G_i$	[a] 0.	[a] +429300.	+1676000.	+3681000.	+6452000.	* * * *	+1824000000.	+205700000.
(j) $\frac{\text{(i)}}{12n^2} = G_i = G_0(\xi)$	0.	+89. 4	+349. 1	+767.	+1344.	* * * *	+380000.	+428500.
$\frac{X_o}{G_o}$		$2.26 \cdot 10^{-3}$	$3.24 \cdot 10^{-3}$	$3.49 \cdot 10^{-3}$	$3.56 \cdot 10^{-3}$	* * * *	$2.14 \cdot 10^{-3}$	$2.12 \cdot 10^{-3}$

[a] Computed by first and second parts of equations 12.

NOTE.—In this type of integration it is sometimes necessary to carry intermediate steps to more significant figures than are retained in the result. To maintain the alignment of the columns zeros are written beyond the figures retained.

Assume that m computations were required to satisfy this criterion. Equation 9 may then be written as

$$X_{m+1}(\xi)=\frac{\rho p^2 L^2 G_m}{E}=X_m(\xi) \tag{14}$$

Solve this for the natural frequency f corresponding to this mode:

$$f=\frac{p}{2\pi}=\frac{1}{2\pi L}\sqrt{\frac{E}{\rho}}\sqrt{\frac{X_m}{G_m}} \tag{15}$$

(c) CALCULATION OF STRESS

Knowing the mode, the stress distribution can be computed, provided the transverse stresses are negligible. In that case the longitudinal stresses are:

$$\sigma_x=E\epsilon_x=Ec\frac{d^2X}{dx^2}=\frac{Ec}{L^2}\frac{d^2X}{d\xi^2} \tag{16}$$

where c is the distance from the neutral axis to the extreme fiber

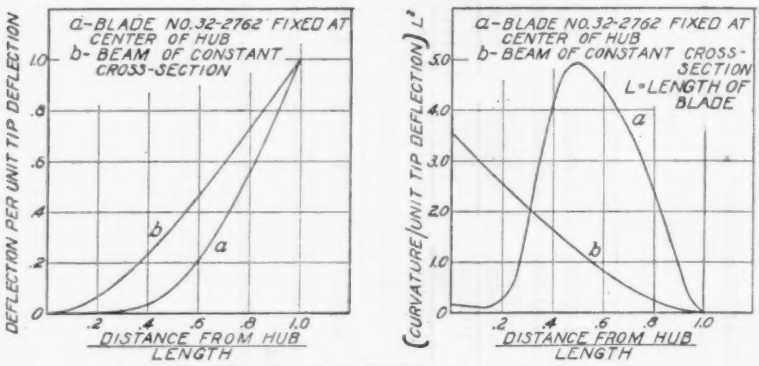

FIGURE 8.—*Fundamental mode and curvature.*

Or, if d is the deflection at the tip, the stress amplitude per unit tip deflection becomes:

$$\frac{\sigma_x}{d}=\frac{Ec}{L^2 d}\frac{d^2X}{d\xi^2} \tag{17}$$

The neglect of transverse stresses in this analysis is probably justified in first approximation; the experimental data show that the transverse stresses are small compared with the longitudinal stresses in the region of maximum stress (see fig. 5, 6). A more exact calculation of the stresses would require a knowledge of ϵ_y as well as ϵ_x which is proportional to the curvature of the mode. This involves a complete solution of the deformation of a solid of variable section under the action of distributed loads. It was thought not worth while to attempt such a solution in view of the extra work involved and the small correction which would result from it.

2. FUNDAMENTAL MODE OF FIXED-FREE PROPELLER BLADE

Curve a, figure 8 (left diagram) shows the fundamental mode for a blade of type 32 computed by the method outlined above. The blade is assumed clamped at the center of the hub. The fundamental mode for a clamped-free bar of constant cross section with unit deflection at the tip [16] is shown for comparison. The two curves differ considerably.

That difference is brought out much more emphatically by intercomparing the curvatures of the neutral fibers for these cases (fig. 8, right diagram). While the maximum curvature for the blade comes at the middle, that for the uniform bar occurs at the point of clamping. It is not safe, therefore, to conclude anything regarding the stresses in propeller blades from the stresses in uniform bars.

The stresses per unit tip deflection may be computed from the curvature by means of eq 17, provided the extreme fiber distance c is known. The distribution of c/L along the centerline for both the convex side and the concave side of a blade of type 32 is shown in figure 9.

Curve a, figure 10, shows the stress distribution for the fundamental mode of blade type 32 clamped at the center of the hub. The Young's modulus E and the specific density of the material (25ST) from which the blades were forged are here taken as

$$E=10^7 \text{ lb/in.}^2$$
$$\rho=2.593\ 10^{-4} \text{ lb-mass/in.}^3$$

The stress distribution differs only little from that observed (curve c, fig. 10). But the frequency which is according to eq 15 equal to

$$f=30.7 \text{ cycles/second}$$

is some 14 percent lower than that observed. This suggests that the point of fixity chosen in the computation should have been placed at a point some distance from the hub and toward the tip. This would shorten the effective length of the blade, and assuming that the blade behaved as a uniform beam in which the frequency is inversely proportional to the square of the length, would lead to a higher frequency. It was decided, therefore, to repeat the computation choosing a point of fixity 15 percent of the distance from the center of the hub to the tip of the blade. This is the point ($\xi=0.15$) which corresponds roughly to the limit of the contact area between hub-clamp and blade. The resulting stress distribution is shown in curve b, figure 10. It is seen to differ only slightly from the stress distribution for clamping at the hub. The frequency becomes

$$f=26.6 \text{ cycles/second}$$

which is a decrease although the effective length of the blade has been decreased. A more detailed investigation shows that this apparent anomaly is due to the rapid decrease in section near the root of the blade. This shows again the need for caution in drawing analogies between a beam of uniform section and one of variable section such as a propeller blade.

[16] Lord Rayleigh. Theory of Sound, par. 173 (Macmillan Co., New York, 1894).

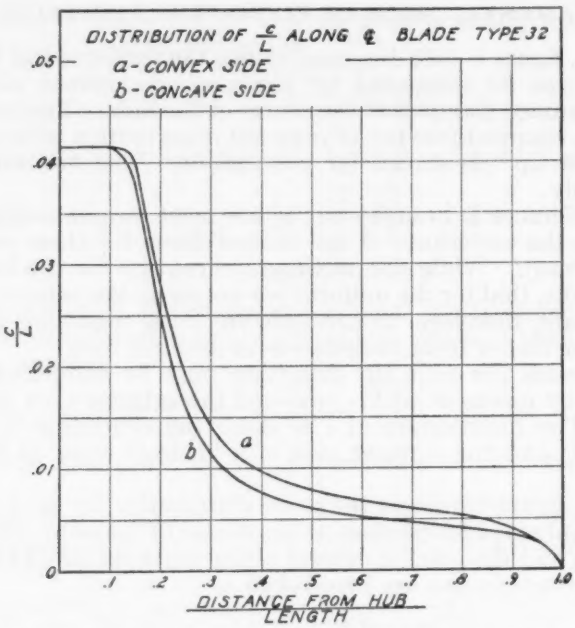

FIGURE 9.—*Variation of distance (c) from neutral axis to extreme fiber along blade.*

(*L* length of blade)

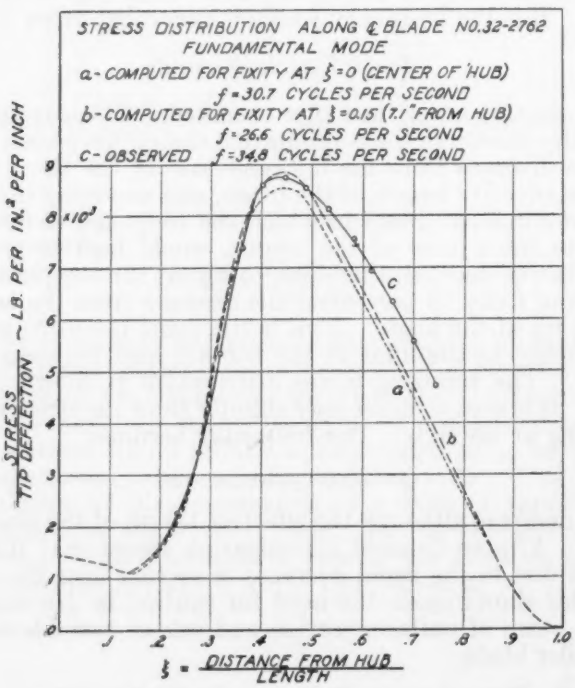

FIGURE 10.—*Stress distribution, fundamental mode.*

3. SECOND HARMONIC MODE OF FIXED-FREE PROPELLER BLADE

The procedure outlined above must be modified in the computation of the second harmonic mode of the fixed-free blade. The correct mode must again satisfy the integral eq 9. In addition it must have a node at a point near the tip of the propeller. Hansen and Mesmer arbitrarily assume the location of this node at a point one-fourth of the blade length in from the tip ($\xi=0.75$) and then proceed to compute the mode for this nodal position by starting with a mode X_0 satisfying the end conditions at the tip and at the hub and also going through

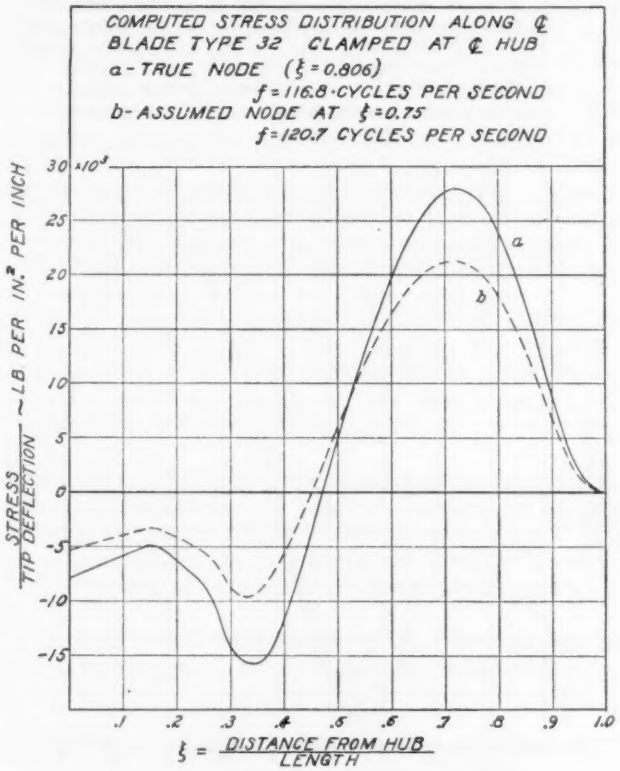

FIGURE 11.—*Stress distribution, second harmonic mode.*

zero at the assumed node. Using this value of X_0 they compute X_1 from eq 9. The mode X_1 will now, in general, have its node at a point some distance away from the assumed point $\xi=0.75$; to pull it back into this point Hansen and Mesmer add a mode proportional to the fundamental mode and choose the factor of proportionality such as to make the resultant mode go through 0 at $\xi=0.75$. If this procedure is repeated a few times the resultant mode will be proportional to the initial mode; Hansen and Mesmer then compute the frequency from their ratio as given by eq 15. It is clear that the condition eq 9 of equality of initial and final mode is not satisfied by this method unless the proportion of fundamental mode to be added is equal to 0. This suggested the following procedure for computing the true mode and correct position of the node.

The ratio of the tip amplitude of the fundamental mode, added for compensation, to the amplitude of the second harmonic mode as computed by eq 9 is plotted for various positions of the node, say $\xi=$ 0.7, 0.75, 0.8, and the nodal position for which this ratio is equal to 0 is obtained from these points by interpolation. Then the computation is repeated for the optimum location of the node to verify that the condition eq 9 actually holds within slide-rule error.

The frequency as computed by Hansen and Mesmer's method of forcing the node at $\xi=0.75$ practically agrees with that computed for

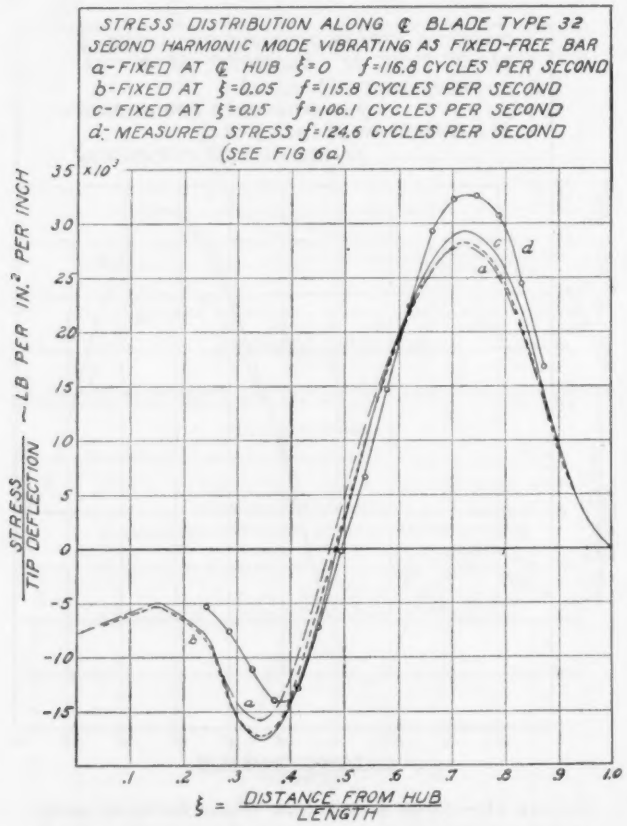

FIGURE 12.—*Stress distribution, second harmonic mode.*

the theoretically correct node, but the stress distribution so computed does not. This point is brought out in figure 11, which shows the stress distribution computed for blade 32–2762 fixed at the hub and vibrating in one case (curve a) with the theoretically correct mode having a node at $\xi=0.806$ and in the other (curve b) with the node at $\xi=0.75$ as assumed by Hansen and Mesmer. It is seen that the stress distributions for the two computations are similar in shape, but the stress per inch tip deflection for the correct node is some 20 to 30 percent higher than that for the node assumed at the ¾ point. The frequencies for these cases agree within 4 percent. It appears desirable, therefore, to find the solution of eq 9 whenever one is

interested in the stress distribution as well as in the frequency of the second harmonic.

The frequency of the second harmonic mode assuming fixity at the center of the hub was calculated to be 116.8 cycles/second against an observed value of 124.6 cycles/second. It was first thought that this divergence might be due to selecting the wrong point of fixity, and hence it was decided to compute the stress distribution and frequency for two other points, one with the blade clamped at $\xi=0.05$ (2.38 in. from the center of the hub) and the other with the blade clamped at $\xi=0.15$ (7.14 in. from the center of the hub). The results of these computations are shown in figure 12. It is seen that the change in point of fixity has had little effect on the stress distribution. The measured stress distribution for blade 32–2758 is shown by curve d (see also fig. 6 (a)). The computed and the observed curves agree within 15 percent except near the tip and near the hub. Near the tip the uncertainties in both the experimental and theoretical results are greatest, in one case because of the severe vibration at that point and in the other because of possible errors in measuring the section of the blade. The discrepancy between observed and calculated stress distribution near the hub may be due to assuming ideal fixity at the hub instead of the true end condition, which must be somewhere between that for a free-free bar and that for a fixed-free bar.

As the point of fixity moves from the center of the hub to a point 15 percent of the blade length toward the tip, the frequency is lowered some 10 percent. One notes again the unusual effect of lowering of frequency with shortening of the free length of the blade, although the effect is not as pronounced as for the fundamental mode.

This discrepancy in natural frequencies cannot be explained by the neglect of shear deformation and rotational inertia in setting up eq 9, since both these factors tend to lower the frequency [17] [18], while the frequencies calculated in the preceding are already lower than those observed. The effect of restraint at the hub remains as a possible explanation.

Hence it seemed worth while to carry out the solution for a blade of design 32 vibrating with its fundamental frequency as a free-free bar.

4. FUNDAMENTAL MODE OF FREE-FREE PROPELLER BLADE

The integral equation for a natural mode satisfying the boundary conditions for a free-free bar:

$$\left.\begin{aligned} &\frac{d}{d\xi}\left(\frac{I}{L^4}\frac{d^2X}{d\xi^2}\right)=0,\ \frac{I}{L^4}\left(\frac{d^2X}{d\xi^2}\right)=0,\ at\ \xi=1 \\ &\frac{dX}{d\xi}=0,\ \frac{d}{d\xi}\left(\frac{I}{L^4}\frac{d^2X}{d\xi^2}\right)=0,\ \text{at}\ \xi=0 \end{aligned}\right\} \tag{18}$$

may be found by integrating eq 7 four times with respect to ξ and each time determining the constant of integration by one of the boundary conditions. The first three boundary conditions eq 18 are the same

[17] Lord Rayleigh. Theory of Sound, par. 186 (Macmillan, New York, 1894).
[18] S. Timoshenko. Phil. Mag. 41, 744 (1921).

as for the clamped-free bar eq 8. The integral expression for the slope dX/dx will, therefore, be the same and that for the mode can differ by a constant only:

$$X=X_1+c_0 \tag{19}$$

where X_1 is the expression on the right side of eq 9 and c_0 may be evaluated by substituting eq 19 in the integral equation for shearing force:

$$\frac{d}{d\xi}\left(\frac{I}{L^4}\frac{d^2X}{d\xi^2}\right)=\frac{\rho p^2L^2}{E}\int_1^\xi \frac{A}{L^2}Xd\xi \tag{20}$$

and making the right side equal to 0 at $\xi=0$. Inserting the value of c_0 so obtained in eq 19 gives the following integral equation for the natural mode of a free-free bar:

$$X=\frac{\rho p^2L^2}{E}\left(\int_0^\xi\int_0^\xi\frac{L^4}{I}\int_1^\xi\int_1^\xi\frac{A}{L^2}Xd\xi d\xi d\xi d\xi-\right.$$
$$\left.-\frac{1}{\int_0^1\frac{A}{L^2}d\xi}\int_0^1\frac{A}{L^2}\int_0^\xi\int_0^\xi\frac{L^4}{I}\int_1^\xi\int_1^\xi\frac{A}{L^2}Xd\xi d\xi d\xi d\xi d\xi\right) \tag{21}$$

The solution of the integral on the right of eq 21 involves double integrals of the form G and F given in eq 10 and in addition single integrals of the form:

$$H(\xi)=\int_0^\xi \chi(\xi)d\xi \tag{22}$$

The integral H may be evaluated numerically in a manner analogous to that used in solving G, F. One chooses an interval spacing $1/n$ that is so close that the value of the integrand over any interval $\frac{i}{n}\geqq\xi\geqq\frac{i+2}{n}$ may be closely approximated by a parabola passing through the 3 points $\chi\left(\frac{i}{n}\right)$, $\chi\left(\frac{i+1}{n}\right)$, $\chi\left(\frac{i+2}{n}\right)$. The value $H(\xi)$ at the end of the first interval and that at the end of the i-th interval are then given by:

$$H_1=\frac{1}{12n}(5\chi_0+8\chi_1-\chi_2)$$
$$H_i=H_{i-1}+\frac{1}{12n}(-\chi_{i-2}+8\chi_{i-1}+5\chi_i) \tag{23}$$

where H_i and χ_i denote values of $H(\xi)$ and $\chi(\xi)$ at the end of the i-th interval $\left(\xi=\frac{i}{n}\right)$. The integration may be conveniently tabulated in a manner analogous to that shown in table 1 for the integration of $F(\xi)$ and $G(\xi)$.

The fundamental mode of a blade, design type 32, vibrating as a free-free bar, was found from eq 21 with the help of such a tabulation using 20 intervals ($n=20$), and then repeating the integration until the ratio $\frac{\rho p^2L^2}{E}$ in eq 21 assumed the same value at the end of each interval.

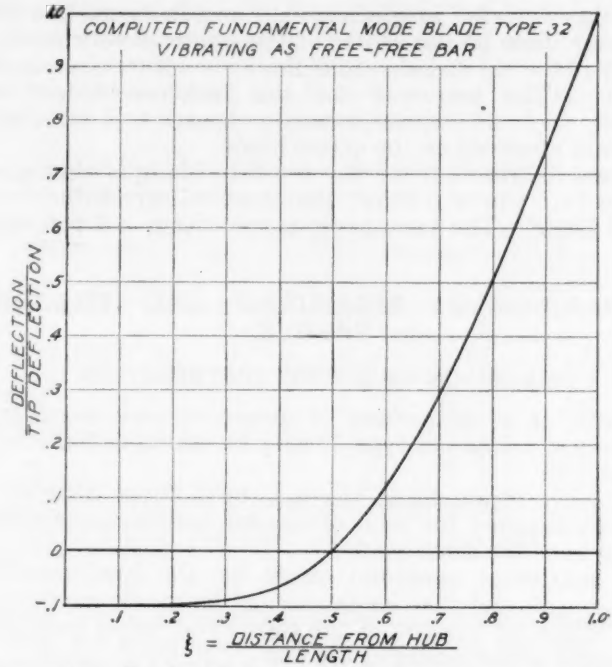

FIGURE 13.—*Fundamental mode, blade vibrating as free-free bar.*

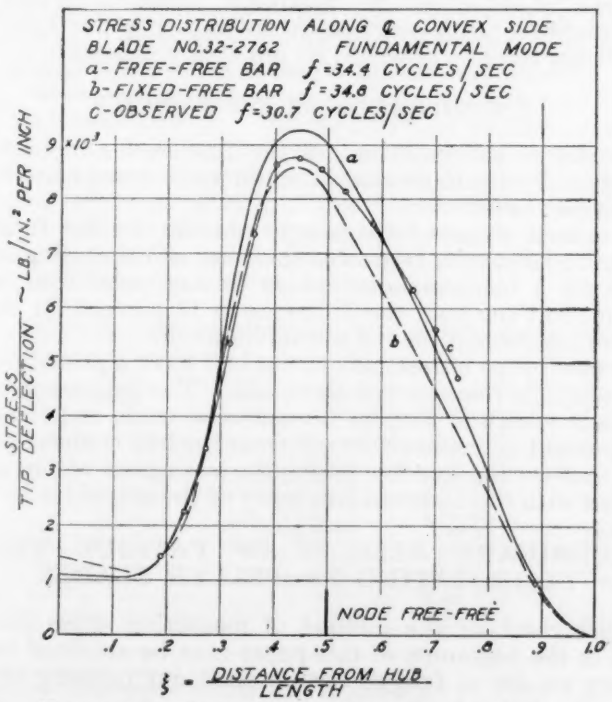

FIGURE 14.—*Stress distribution, fundamental mode.*

The mode calculated in this way is shown in figure 13. It is seen that the amplitude at the center of the hub is closely equal to one-tenth that at the tip and also that the node occurs close to the position $\xi=0.5$. The frequency for the free-free blade comes out theoretically as $f=34.4$ cycles/second compared to a value of $f=34.8$ cycles/second observed on the actual blade.

The stress distribution for the free-free blade is shown as curve a in figure 14. Curve c shows the observed stress distribution for the actual blade. The two curves agree within ± 8 percent for the most part.

V. COMPARISON OF MEASURED AND THEORETICAL VALUES

1. COMPARISON OF STRESS DISTRIBUTION

The result of a comparison of measured and calculated stress distributions of a blade of type 32 may be stated as follows (see figs. 10, 12, and 14):

a. The curve representing the measured stress distribution and the curves computed for each of several conditions of restraint at the hub all have the same shape.

b. The maximum measured stress for the fundamental agrees within 3 percent, and that for the second harmonic mode (with node near tip) within 15 percent of the theoretical stress computed for a blade clamped at the center of the hub.

c. The conditions of clamping have only slight effect on the theoretical stress distribution, the maximum stress calculated for the extreme condition of no restraint at the hub (free-free bar) differing from the stress actually observed by not over 7 percent.

2. COMPARISON OF FREQUENCIES

The results of a comparison of the frequencies measured for a blade of type 32 with those calculated for various end conditions may be summarized as follows:

a. The actual observed frequency, whether for the fundamental or the second harmonic, is greater than the corresponding frequency computed for a blade clamped at, or at any point near, the hub. For clamping at the hub, the difference is 14 percent for the fundamental and 6.5 percent for the second harmonic.

b. The conditions of restraint at the hub have a pronounced effect on the theoretical frequency of the blade. The frequency calculated for the blade vibrating with its fundamental mode and clamped at a point 15 percent of the blade length from the hub is about 24 percent less than that for the free-free blade; the last agrees within a fraction of 1 percent with the observed frequency of the actual blade.

VI. PRELIMINARY RESULTS OF FATIGUE TESTS ON NONROTATING PROPELLER BLADES

A further check of the method of measuring stress distribution described in the beginning of this paper may be obtained from some preliminary results of fatigue tests carried out recently at the National Bureau of Standards.

Eight propeller blades of various designs, forged from 25ST aluminum alloy, were vibrated at their fundamental mode, and explored with a strain gage. Two of the blades (nos. 43,903 and 43,904) were new, and the remaining six had been in service. Table 2 gives the magnitude and location of maximum strain.

TABLE 2.—*Characteristics of aluminum alloy blades*

Blade number [1]	Blade length	Frequency [2]	Maximum longitudinal strain amplitude for 1-in. tip amplitude	Distance of point of maximum strain from tip
	in.	*Cycles/second*	10^{-4} in.	in.
33,155 (4)	48	31.7	7.30	28½
25,460 (1)	48	33.1	6.75	24½
25,363 (099)	48½	32.5	6.25	24
25,099 (363 and 377)	48	32.6	6.76	24
43,904 (3) [4]	48	33.8	6.55	24
43,903 (4 and 26,681) [4]	48	33.8	7.10	22¾
25,626 (7)	46½	36.0	7.80	20½
27,056 (7)	49½	30.7	[3] (6.25)	31

[1] The last digit, or digits, of the number identifying the second blade of the propeller are enclosed in parentheses. In two cases the second blade was changed during test.
[2] All frequencies on 15 hp motor. Frequency of 33,155 on 7½ hp motor was 32.4.
[3] Measured on blade 27,057 only.
[4] New blade.

The blades were then subjected to high alternating bending stresses by increasing the armature current of the motor, M, (figs. 1, 2) until the tip amplitude reached a value corresponding to the desired stress amplitude. For the first two propellers tested, the stress amplitude had been chosen as 12,700 and 15,400 lb/in.^{2}, values which proved to be too small to produce fatigue failure after more than a million cycles. The tip amplitudes at which these two blades were run were, therefore, raised to values corresponding to a maximum stress of 24,100 and 19,900 lb/in.^{2}, respectively. The blades failed at these stresses after about two hours running for the first propeller and less than 20 minutes for the second propeller. The remaining six blades were run at one stress only (see table 3). The final fracture was preceded in every case by the formation of a typical fatigue crack across the blade at a section in the middle third where the stress differed only a few percent from the measured maximum stress.

TABLE 3.—*Data on fatigue failures of aluminum alloy blades*

Blade number	Maximum stress	Number of cycles	Distance of crack from tip	Stress at crack
	lb/in.^{2}		in.	lb/in.^{2}
33,155	12,700 for	1,350,000		
	24,100 for	214,000	25	[1] 23,000
25,460	15,400 for	2,140,000		
	19,900 for	33,800	24½	[2] 19,700
25,363	16,700	505,000	19¾, 20½, 21	15,500
25,099	14,500	442,000	23	14,000
43,904 [2]	17,500	2,060,000	27½	17,200
43,903 [2]	16,600	2,390,000	29	16,100
25,626	21,400	233,000	24½	19,500
27,056	18,800	379,000	27½	18,100

[1] In the preceding run the stress at this point was only 12,200 lb/in.^{2}
[2] In the preceding run the stress at this point was only 15,300 lb/in.^{2}
[3] New blade.

The number of cycles to failure are plotted against maximum stress on the S–N diagram of figure 15. This same figure shows for comparison one S–N diagram (for strips of 25ST material) obtained by Briggs and Murphy in an air-driven high-speed fatigue machine at the National Bureau of Standards and one (for rotating cantilever beam specimens of the same alloy) obtained by D. J. McAdam, Jr.[19] Finally it shows the S–N diagram for fatigue tests on specimens cut from a 25ST propeller blade.[20] The eight points for the failure of full-size blades fall, for the most part, below the S–N diagrams for polished specimens. The points scatter considerably, and are too few in number to give a reliable value for the fatigue limit of a full-size blade. Much of the observed scatter may be due to differences in surface

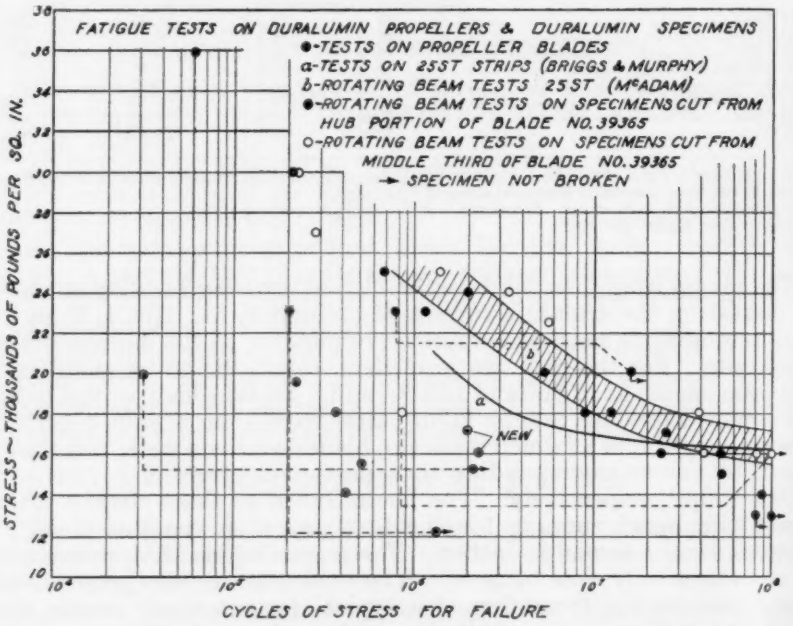
FATIGUE TESTS ON DURALUMIN PROPELLERS & DURALUMIN SPECIMENS
●-TESTS ON PROPELLER BLADES
a-TESTS ON 25ST STRIPS (BRIGGS & MURPHY)
b-ROTATING BEAM TESTS 25ST (McADAM)
●-ROTATING BEAM TESTS ON SPECIMENS CUT FROM HUB PORTION OF BLADE NO.39365
○-ROTATING BEAM TESTS ON SPECIMENS CUT FROM MIDDLE THIRD OF BLADE NO.39365
→ SPECIMEN NOT BROKEN
NEW
STRESS—THOUSANDS OF POUNDS PER SQ. IN.
CYCLES OF STRESS FOR FAILURE

FIGURE 15.—*Results of fatigue tests on duralumin propellers and duralumin specimens.*

condition of the blades. It will be seen that the new blades tested under the same conditions as the used blades withstood a greater number of alternations before failure. One failure on a used blade (no. 25,460) was noticed in the earliest stages and the crack was observed to follow a tortuous curve from one small pit in the surface to another. The highly polished surface of the new propellers was relatively free from irregularities.

VII. CONCLUSIONS

1. By means of the experimental method described in this paper the stress distribution for the fundamental mode and for the second harmonic mode (with node near tip) of a duralumin blade type 32

[19] Pamphlet no. 1,537–D, issued with Mining and Metallurgy (Feb. 1926).
[20] These tests were made by W. H. Swanger and R. D. France of this Bureau.

can be determined; in the region of maximum stress it agrees within 12 percent with the computed distribution.

2. The theoretical stresses for the fundamental mode are, within 10 percent, independent of the restraint at the hub.

3. The frequency is much more sensitive to the restraint at the hub than the stress distribution. It changed by some 26 percent for a change in restraint that produced a variation of only 10 percent in maximum stress for the fundamental.

4. Artificially produced fatigue failures on eight propeller blades occurred in each case at a point where the stress was within 10 percent of the maximum stress measured on the blade.

The third of these conclusions emphasizes the fact that computation, while giving good approximations for the stresses, is not, in general, a reliable means for determining the natural frequencies of a propeller blade; it is safer to rely on direct measurements of resonant frequencies insofar as they are obtainable. One step towards obtaining such measurements is the propeller-vibration indicator developed by Dryden and Tuckerman [21] in connection with this research program.

WASHINGTON, December 18, 1934.

[21] BS J. Research **12**, 537 (1934) RP678.

U. S. Department of Commerce National Bureau of Standards

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COMBINING WEIGHT OF COLLAGEN

By John Beek, Jr.

ABSTRACT

The adsorption of hydrochloric acid by collagen was determined in solutions varying in acid concentration from 0 to about 1.7 molal. The solutions were nearly or completely saturated with sodium chloride. The high concentration of salt was used to repress the swelling of the collagen, and to provide a reference substance for the measurement of the adsorption. The quantity of acid reacting with the collagen was found to vary continuously with the concentration of acid. The nature of the relation between the concentration and the quantity of acid combined indicates that at least 3 types of compounds are formed by the collagen and the hydrochloric acid. At low concentrations, the combining weight of the collagen is about 1,000. It is deduced that 3 nitrogen atoms in each unit of 38 are contained in strongly basic groups. The equivalent weight of collagen is indeterminate under the conditions of this experiment. So many types of nitrogen-containing groups are reacting with the acid that not even an approximate value of the true equivalent weight can be selected.

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I. INTRODUCTION

Determination in aqueous solution of the amount of acid which combines with collagen presents a serious difficulty. The collagen swells, taking up solution amounting to from six to eleven times its dry weight. It has been shown by Procter (1) [1] that concentration of acid in the gel phase is less than that in the free solution, the only condition being that a partially ionized salt be formed. No precise correction for the consequent error is possible in the present state of knowledge of the system.

This difficulty may be partly overcome by adding salt to the solution. This decreases swelling and tends to equalize the concentration of acid in the two phases. However, there is a question as to how closely the behavior of the collagen in the salt solution follows that in a solution free from salt.

[1] The figures, 1 to 10, given in parentheses here and elsewhere in the text correspond to the numbered references at the end of this paper.

V. Kubelka (2) measured the adsorption of hydrochloric acid on hide powder in 10 percent, 20 percent, and saturated (26 percent) solutions of sodium chloride. In the case of the saturated solutions, a remarkable relationship between concentration of acid and the amount of acid adsorbed was indicated. The curve showing the relation of adsorption to concentration exhibited a sharp break at about one-tenth normal and an adsorption of about 1.03 millimoles of acid per gram of dry hide powder. As there were only 12 determinations in the concentration range up to approximately normal acid, the form of the curve was not well defined. The purpose of the present work was to determine the nature of the relation between the adsorption and the concentration of acid in solutions nearly saturated with sodium chloride, with the object of determining the combining weight of collagen with respect to acid in dilute solution.

II. EXPERIMENTAL MATERIAL AND METHOD

The collagen was prepared from limed and unhaired steer hide. After deliming and bating, the hide was washed with distilled water, and dried by washing first with 95 percent ethyl alcohol and then with xylene. After drying, the grain and flesh surfaces were split off, and the remaining collagen was cut into narrow strips about 3 cm long. This material was prepared for sampling by conditioning it in an atmosphere in equilibrium with a solution saturated with sodium chloride.

Most of the determinations were made in solutions which were only 93 to 99 percent saturated with sodium chloride. Thirty determinations at low acid concentrations were made in solutions saturated with sodium chloride.

Approximately 5 g of collagen was used with about 50 ml of solution. One day was allowed for the system to approach equilibrium, a series of determinations having shown that equilibrium with respect to the adsorption was practically attained in this time. The final acid concentration was measured by pouring the solution into a beaker and titrating it, the weight of the solution poured off being measured by reweighing the flask and its contents. An excess of sodium hydroxide solution was added, which was then titrated with dilute hydrochloric acid. With the exception of the titration of this excess, all standard solutions were measured by weight. In case the solution was not saturated with sodium chloride, its concentration was determined by evaporating to dryness and heating the residue at 150° C. The weight of this residue was corrected for the collagen present, and for sodium chloride equivalent to the sodium hydroxide added in the titration. An additional correction of 0.27 percent was necessary because the sodium chloride retained that much water under these conditions. In each case the total nitrogen in the solution poured off was determined by the Kjeldahl method. This quantity was always small, exceeding the equivalent of 15 mg of collagen in only four determinations at the highest acid concentrations.

In 15 determinations the reversibility of the reaction was tested by adding about 45 ml of a nearly saturated solution of sodium chloride

to the collagen and solution remaining in the flask after the completion of a regular determination. The procedure of the regular determination was then repeated, using the known weight and concentration of the solution added to calculate the amount of sodium chloride in the system. These determinations will be referred to as "reversed" in the text and in figure 3.

In the determinations at the four highest acid concentrations a considerable hydrolysis of the collagen took place. The average collagen equivalent of the nitrogen in the solution poured off was 84 mg for these four while for the four determinations next lower in acid concentration the corresponding average was 8 mg. The presence of the dissolved products of hydrolysis interfered somewhat with

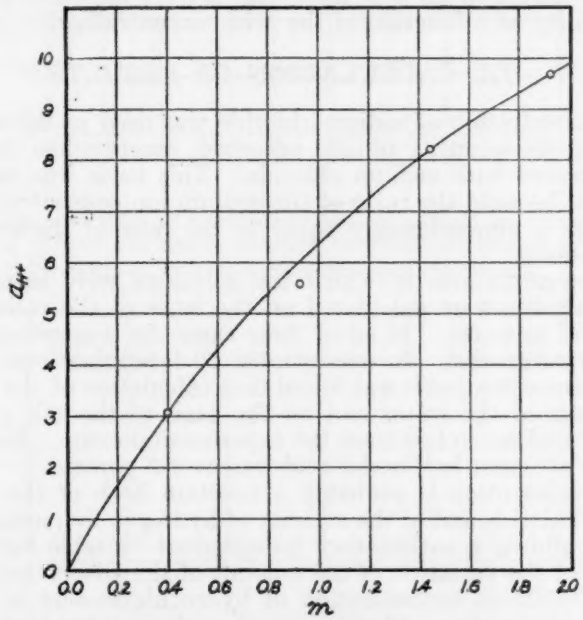

FIGURE 1.—*The activity of hydrogen ion in solutions of hydrochloric acid saturated with sodium chloride at 25° C.*

m is the molality of hydrochloric acid.

the titration. A more serious error arose from the failure of the system to reach equilibrium with respect to the distribution of the products of hydrolysis, apparently because the rate of hydrolysis was greater than the rate of diffusion of the products. For the determinations that were reversed from these four the average value of the collagen equivalent of the nitrogen in the solution poured off was 26 mg, while the average for the other determinations in the same range of concentration was 6 mg. The excess of hydrolytic products in the collagen phase makes the calculated value of the adsorption too high.

The hydrogen ion activities in a series of solutions of hydrochloric acid saturated with sodium chloride were measured with the hydrogen electrode. The results are shown in figure 1, where a_{H^+} repre-

sents the activity of hydrogen ion and m is the molality of hydrochloric acid.

The curve for interpolation was selected so as to be consistent with the results of Thomas and Baldwin (3) at low concentrations of acid. Activities of hydrogen ion in the experimental solutions were calculated from these data and those of H. C. Jones (4), assuming the linearity of the relationb etween pH and molarity of sodium chloride at constant molarity of acid.

Data presented by J. Erskine Hawkins (5) were used to calculate the activity of hydrochloric acid.

The solubility at 25° C of sodium chloride in hydrochloric acid solution was determined at four acid concentrations. Values at intermediate concentrations were interpolated from the curve showing the total molality as a function of the acid concentration.

III. CALCULATION OF RESULTS

The concentration of sodium chloride was used as the reference in calculating the quantity of acid adsorbed, except when the solutions were saturated with sodium chloride. This basis was used for the calculation because the ratio of the sodium ion concentrations in the two phases is approximately equal to the ratio of the hydrogen-ion concentrations.

The determinations in which the solutions were saturated with sodium chloride were calculated on the basis of the ratio of hydrochloric acid to water. In all of these cases the concentration of acid was below 0.05 molal. In one series of 10 determinations in the same range of concentration it was found that calculation of the adsorption on the basis of the water and on the basis of the salt gave results which differed much less than the experimental error. However, the difference at about half molal acid was over 3 percent.

As the adsorption is probably a function both of the activity of hydrogen chloride and of the activity of hydrogen ion, neither of these activities affords a satisfactory independent variable for the representation of the variation of the amount of the adsorption. For this reason a fictitious concentration of hydrochloric acid in a solution saturated with sodium chloride is used as the independent variable in presenting the data. This concentration is calculated as that giving the same activity of hydrogen chloride as that in the actual solution. This calculated concentration is designated as m. Concentrations of this type calculated from the activity of hydrogen chloride and from that of hydrogen ion are in close agreement at concentrations below 1 molal.

The quantity used to present the results is the fraction of the total nitrogen which reacts with the acid. This is calculated as the ratio of the number of millimoles of acid adsorbed to the number of milligram atoms of nitrogen in the sample of collagen. This fraction is designated by the letter f.

Figures 2, 3, and 4, show the experimental results. Some of the data at low acid concentrations are omitted from figures 2 and 3 because of the small scale of these figures.

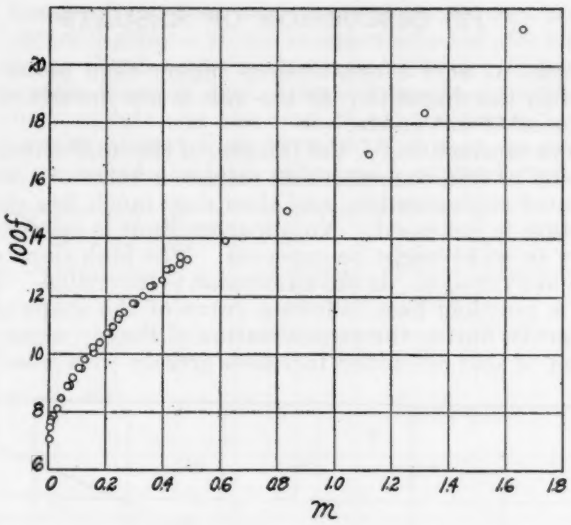

FIGURE 2.—*The adsorption of hydrochloric acid by collagen as a function of the molality of hydrochloric acid in solutions saturated with sodium chloride.*

f is the fraction of the total nitrogen which reacts with the acid.

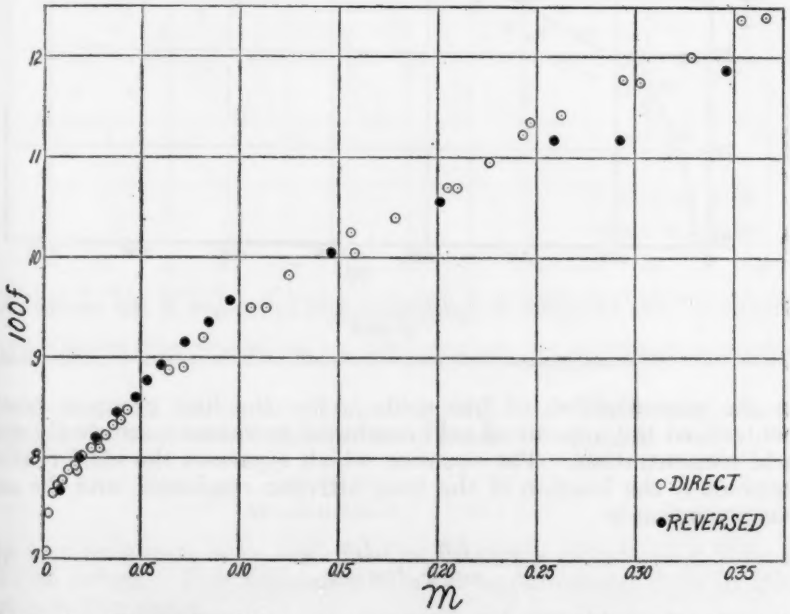

FIGURE 3.—*The results of the experiment testing the reversibility of the adsorption of hydrochloric acid by collagen in solutions saturated with sodium chloride.*

f is the fraction of the total nitrogen which reacts with the acid. *m* is the molality of hydrochloric acid.

IV. DISCUSSION OF RESULTS

The results at acid concentrations higher than molal will not be considered in the discussion, as the values are uncertain, because of the error pointed out above.

The curve representing f, the fraction of the total nitrogen reacting, as a function of acid concentration reaches a value of about 0.08 at a very low acid concentration, and then rises much less rapidly as the concentration is increased. No apparent limit is approached, which is contrary to what might be expected. The high slope of the curve is not due to hydrolysis, as the adsorption is reversible. The neutralization of a polyacid base follows a curve of the shape given by the data. That is, during the neutralization of the strongest basic group the amount of acid combined increases greatly with small increments

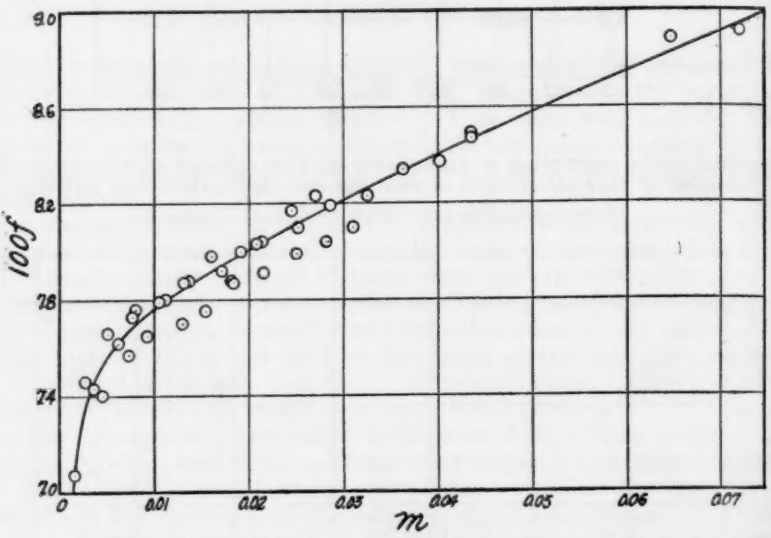

FIGURE 4.—*The adsorption of hydrochloric acid by collagen at low concentrations of acid.*

f is the fraction of the total nitrogen which reacts with the acid. m is the molality of hydrochloric acid.

in the concentration of free acid. After the first group is nearly neutralized the amount of acid combined increases more slowly with acid concentration. The equation which expresses the ideal relation between f, the fraction of the total nitrogen combined, and the acid concentration is

$$f=\frac{f_1 m}{m+k_1}+\frac{f_2 m}{m+k_2}+\cdots, \tag{1}$$

where f_1, f_2, etc., are the fractions of the total nitrogen contained in basic groups which form salts having the hydrolysis constants k_1, k_2, etc., respectively. Equation 1 is hardly better than a rough approximation, because the variations of the activity coefficient of the acid and the ionization of the collagen salts formed are not taken into account. However, the constants in the numerators have a precise significance, and it will be shown that one of them can be evaluated.

A plot of the results at low acid concentrations on a scale larger than that of figure 2 gives a better representation of the form of this part of the curve. Such a plot is shown in figure 4. It may be seen that on this scale there is very little curvature at concentrations higher than 0.01 molal. As the sum of all the terms beyond the first in eq 1 varies almost linearly with m it may be represented accurately by a term of the form $m/(a+bm)$, where a and b are em-

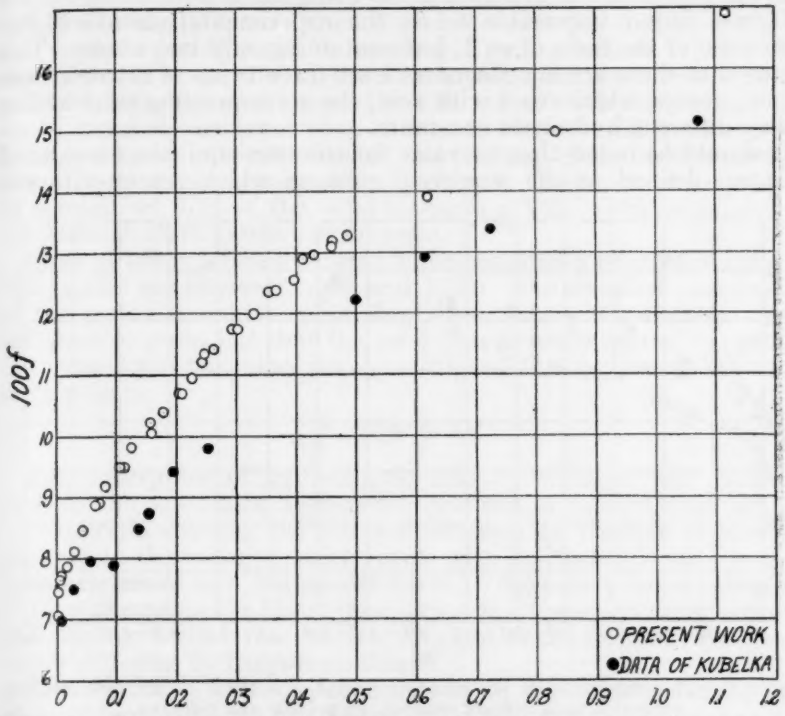

FIGURE 5.—*The adsorption of hydrochloric acid by collagen in solutions saturated with sodium chloride.*

The data of Kubelka (2) are compared with those of the present work. (f is the fraction of the total nitrogen which reacts with the acid. m is the molality of hydrochloric acid).

pirical constants having no physical significance. The smooth curve of figure 4 represents values calculated from the equation

$$f=\frac{0.0766\ m}{m+0.000136}+\frac{m}{11\ m+4.77}. \qquad (2)$$

The four constants were evaluated to make the calculated f fit four selected points. This expression fits the experimental data in this concentration range.

The constant 0.0766 in eq 2 represents the fraction of the total nitrogen which occurs in groups which are much more strongly basic than others in the collagen. It has been shown (6) that the number of nitrogen atoms in a structural unit of collagen is 38 or a multiple thereof. The constant 0.0766 is equal to 2.91/38. Thus it may be concluded that 3 nitrogen atoms of each group of 38 are involved in this first part of the neutralization.

One effect of the predominance of these 3 nitrogen atoms in the reaction with acid in dilute solution is to give the collagen an apparent equivalent weight of about 1,000 in dilute solution. This results from the fact that collagen contains about 17.9 percent of nitrogen, so that 78 g of collagen contains 1 g atom of nitrogen. Then the combining weight is (38/3)(78), or about 1,000.

More accurately, the combining weight is (177.4)(100/P), where P is the percentage of nitrogen in the collagen.

It was found impossible to fit the experimental data with an expression of the form of eq 1, but containing only two terms. This means that there are in collagen at least three types of nitrogen-containing groups which react with acid, the corresponding salts having widely different hydrolysis constants.

It should be noted that no value for the true equivalent weight of collagen, defined as the weight of collagen which reacts with one

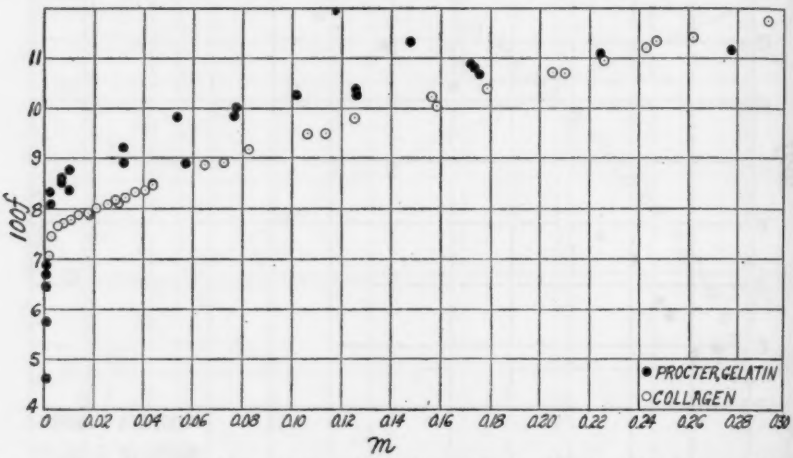

FIGURE 6.—*A comparison of the adsorption of hydrochloric acid, from solutions saturated with sodium chloride, by gelatin and collagen.*

The data for gelatin are taken from the work of Procter (1), and the data for collagen from the present work. f is the fraction of the total nitrogen which reacts with the acid. m is the molality of hydrochloric acid.

equivalent of acid at an indefinitely high concentration, can be derived from these data.

Figure 5 compares Kubelka's (2) results for hide powder with the present work. It does not seem probable that the apparent break exhibited in the graph of Kubelka's results is real. However, it is of interest to note that Kubelka offers a value of 977 for the combining weight of dry hide powder, deriving this figure from the amount of adsorption at the point where the break in his curve occurs.

Procter (1) measured the adsorption of hydrochloric acid by gelatin in solutions saturated with sodium chloride. Atkin and Douglas (7) criticized Procter's method of determining the total acid in the gelatin, pointing out that excess base was required to make the solution of gelatin alkaline to phenolphthalein. The result of this error was to give high values for the amount of acid combined with the gelatin. However, the correction suggested by Atkin and Douglas can hardly be applied quantitatively under the conditions of Procter's

experiment. Figure 6 shows Procter's results, calculated on the basis of 17.9 percent of nitrogen in the dry gelatin, compared with the present work on collagen.

Procter and Wilson (8) used Procter's data to derive an equivalent weight of 768 for gelatin. This value is open to doubt, because of the experimental error pointed out by Atkin and Douglas, and because the amount of acid combined was limited by the range of acid concentration covered in the experiments.

Kubelka and Wagner (9) measured the adsorption of hydrochloric acid by hide powder, and its swelling in acid solutions. From their data Beek (10) calculated the amount of acid combined with the collagen, and derived a value of 969 for the equivalent weight. This would naturally be lower than the value found in the present work, because no account was taken of the reaction of the acid with more than 1 type of nitrogen-containing group. The value corresponding to reaction of 3/38 of the total nitrogen is 993, when calculated on the basis of 17.86 percent of nitrogen.

Most of the published values for the combining weights of collagen and gelatin are between 1,050 and 1,150. No detailed examination of the methods used in obtaining these values is necessary. It is sufficient to point out that the error due to ionization of the protein salt formed was not taken into account, and that as a result the values are too high.

V. SUMMARY

The adsorption of hydrochloric acid by collagen was measured in solutions containing high concentrations of sodium chloride.

The graph showing the relation between the fraction of the total nitrogen in the collagen which reacts with hydrochloric acid, and the concentration of acid, is a smooth curve. The adsorption is reversible.

There is a series of at least three types of nitrogen-containing groups in the collagen which react with hydrochloric acid, forming salts having widely differing hydrolysis constants.

Three nitrogen atoms in each unit of 38 are in groups much more strongly basic than the remainder. Collagen has an apparent equivalent weight of about 1,000 in dilute acid solution.

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